Advanced CSP Teaching Materials

Chapter 2
Solar Radiation

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<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>(A_{ab})</td>
<td>absorber aperture area</td>
<td>(m^2)</td>
</tr>
<tr>
<td>(A_{ap})</td>
<td>aperture area</td>
<td>(m^2)</td>
</tr>
<tr>
<td>(A_{im})</td>
<td>area of the Sun image</td>
<td>m</td>
</tr>
<tr>
<td>(A_S)</td>
<td>surface area of the Sun</td>
<td>(m^2)</td>
</tr>
<tr>
<td>(AM)</td>
<td>Air Mass (relative optical path length of the Sun beams through the atmosphere)</td>
<td>-</td>
</tr>
<tr>
<td>b</td>
<td>Wien proportionality factor</td>
<td>mK</td>
</tr>
<tr>
<td>c</td>
<td>light velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>C</td>
<td>concentration ratio</td>
<td>-</td>
</tr>
<tr>
<td>(C_{max})</td>
<td>maximal concentration ratio</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>Diameter of a paraboloid concentrator</td>
<td>m</td>
</tr>
<tr>
<td>DoY</td>
<td>day of the year</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>equation of time (difference between local mean time and solar time)</td>
<td>min</td>
</tr>
<tr>
<td>f</td>
<td>focal length</td>
<td>m</td>
</tr>
<tr>
<td>G</td>
<td>global irradiance</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{clear})</td>
<td>global irradiance at clear sky conditions</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_b)</td>
<td>direct irradiance</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{b,clear})</td>
<td>direct irradiance at clear sky conditions</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{bn})</td>
<td>direct normal irradiance</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{bn,clear})</td>
<td>direct normal irradiance at clear sky conditions</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{dt})</td>
<td>direct irradiance on a tilted plane</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{d,clear})</td>
<td>diffuse irradiance at clear sky conditions</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_d)</td>
<td>diffuse irradiance</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{dt})</td>
<td>diffuse irradiance on a tilted plane</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{on})</td>
<td>solar irradiance outside the atmosphere for a given day of the year</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_r)</td>
<td>irradiance due to reflected radiation</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{rt})</td>
<td>irradiance on tilted surface due to reflected radiation</td>
<td>W/m²</td>
</tr>
<tr>
<td>(G_{SC})</td>
<td>solar constant (1367 W/m²)</td>
<td>W/m²</td>
</tr>
<tr>
<td>(\frac{G_E}{G_{SC}})</td>
<td>average irradiance at the Earth</td>
<td>W/m²</td>
</tr>
<tr>
<td>h</td>
<td>Planck constant</td>
<td>Js</td>
</tr>
<tr>
<td>h</td>
<td>altitude (above sea level)</td>
<td>m</td>
</tr>
<tr>
<td>HoD</td>
<td>hour of day</td>
<td>-</td>
</tr>
<tr>
<td>k</td>
<td>Boltzmann constant</td>
<td>J/K</td>
</tr>
<tr>
<td>K</td>
<td>clearness index (ratio (G) to (G_{SC}))</td>
<td>-</td>
</tr>
<tr>
<td>(k_t)</td>
<td>clearness index (ratio of actual global irradiance to clear sky global irradiance)</td>
<td>-</td>
</tr>
<tr>
<td>l</td>
<td>Length of a parabolic trough</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>longitude</td>
<td>°</td>
</tr>
<tr>
<td>(L_{loc})</td>
<td>local longitude</td>
<td>°</td>
</tr>
<tr>
<td>(L_T)</td>
<td>time zone reference longitude</td>
<td>°</td>
</tr>
<tr>
<td>m</td>
<td>mass</td>
<td>kg</td>
</tr>
<tr>
<td>(M(\lambda, T))</td>
<td>hemispherical emittance per unit surface area and per unit wave length</td>
<td>-</td>
</tr>
<tr>
<td>n</td>
<td>cloud index</td>
<td>-</td>
</tr>
<tr>
<td>(P_{ab})</td>
<td>radiant power of the absorber</td>
<td>W</td>
</tr>
<tr>
<td>(P_S)</td>
<td>radiation power of the Sun</td>
<td>W</td>
</tr>
<tr>
<td>(Q_E)</td>
<td>Solar energy that reaches the Earth (in one year)</td>
<td>kWh</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$\dot{Q}_E$</td>
<td>total solar radiant power that reaches the Earth</td>
<td>W</td>
</tr>
<tr>
<td>$r_{SE}$</td>
<td>Sun-Earth distance</td>
<td>m</td>
</tr>
<tr>
<td>$r_E$</td>
<td>mean Earth radius</td>
<td>m</td>
</tr>
<tr>
<td>$r_r$</td>
<td>distance between focal point and mirror/mirror rim</td>
<td>m</td>
</tr>
<tr>
<td>$r_S$</td>
<td>Sun radius</td>
<td>m</td>
</tr>
<tr>
<td>$s$</td>
<td>tracking angle (collector zenith angle)</td>
<td>°</td>
</tr>
<tr>
<td>$S$</td>
<td>sensitivity of the radiometer (pyranometer)</td>
<td>V/W/m²</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
<td>K</td>
</tr>
<tr>
<td>$T_{ab}$</td>
<td>absorber temperature</td>
<td>K, °C</td>
</tr>
<tr>
<td>$T_L$</td>
<td>(Linke) turbidity factor</td>
<td>-</td>
</tr>
<tr>
<td>$T_S$</td>
<td>surface temperature of the Sun</td>
<td>K, °C</td>
</tr>
<tr>
<td>$t_{day}$</td>
<td>day length (from sunrise to sunset)</td>
<td>h</td>
</tr>
<tr>
<td>$t_{so}$</td>
<td>solar time</td>
<td>h:min:s</td>
</tr>
<tr>
<td>$t_{std}$</td>
<td>standard time</td>
<td>h:min:s</td>
</tr>
<tr>
<td>$U$</td>
<td>voltage</td>
<td>V</td>
</tr>
<tr>
<td>$x$</td>
<td>extinction coefficient</td>
<td>1/m</td>
</tr>
</tbody>
</table>

**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>altitude angle</td>
<td>°</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>sun beam angle</td>
<td>°, rad</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>solar altitude angle</td>
<td>°</td>
</tr>
<tr>
<td>$\alpha_{hs}$</td>
<td>altitude angle of heliostat normal</td>
<td>°</td>
</tr>
<tr>
<td>$\alpha_{st}$</td>
<td>altitude angle of the solar tower receiver from the heliostat position</td>
<td>°</td>
</tr>
<tr>
<td>$\beta$</td>
<td>tilt angle</td>
<td>°</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>azimuth angle</td>
<td>°</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>solar azimuth angle</td>
<td>°</td>
</tr>
<tr>
<td>$\gamma_{hs}$</td>
<td>azimuth angle of heliostat normal</td>
<td>°</td>
</tr>
<tr>
<td>$\gamma_{st}$</td>
<td>azimuth angle of the solar tower from the heliostat position</td>
<td>°</td>
</tr>
<tr>
<td>$\delta$</td>
<td>declination</td>
<td>°</td>
</tr>
<tr>
<td>$\theta$</td>
<td>incidence angle</td>
<td>°</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>solar zenith angle</td>
<td>°</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
<td>µm</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>wavelength at spectral radiant emittance peak</td>
<td>µm</td>
</tr>
<tr>
<td>$\rho$</td>
<td>reflectivity</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{max}$</td>
<td>maximum albedo</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{min}$</td>
<td>minimum albedo (= ground albedo)</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant</td>
<td>W/(m²K⁴)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>transmission coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_a$</td>
<td>atmospheric transmission factor considering absorption and scattering by aerosol particles</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_{ab}$</td>
<td>atmospheric transmission factor considering absorption</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_g$</td>
<td>atmospheric transmission factor considering absorption by different gases</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_{ms}$</td>
<td>atmospheric transmission factor considering Mie-scattering</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_o$</td>
<td>atmospheric transmission factor considering</td>
<td>-</td>
</tr>
</tbody>
</table>
absorption by ozone

$\tau_{rs}$ atmospheric transmission factor considering Rayleigh scattering

$\tau_w$ atmospheric transmission factor considering absorption by water vapour

$\Phi$ geographic latitude

$\psi$ rim angle

$\omega$ hour angle

$\omega_{ss}$ hour angle of sunrise and sunset

**Acronyms**

AM Air Mass

AU astronomical unit
Summary

In the course of this chapter the physical basics of our main source of energy, the Sun, are presented. The Sun structure and its radiation will be explained. The radiation extinction processes on the way to the Earth’s surface will be discussed. The solar radiation that reaches the Earth’s surface is the fundamental resource for the operation of a concentrating solar power plant. A full understanding of the characteristics of the direct solar radiation is essential for the successful design of a concentrating solar power plant. Basic aspects of the Sun position determination, mirror tracking and radiation concentration are discussed. In the last section, some possibilities for the measurement of the available solar radiation are presented.

Key questions

• Where does the energy for the operation of CSP plants come from?
• Which hurdles do the sunbeams have to overcome on the way to Earth?
• How can we determine the direction of solar radiation?
• How can solar collectors follow the direct solar radiation?
• How can we concentrate solar radiation?
• How can we measure the solar radiation?
1 The origin of the energy for the operation of CSP plants

1.1 The Sun and its structure

The source of the energy we make use of in CSP plants is the Sun, the star at the centre of our Solar System. It has a diameter of about 1,392,000 km (about 109 Earths) and a mass of about $2 \cdot 10^{30}$ kg (about 330,000 times the mass of the Earth). The Sun has a layered structure, as illustrated in the following figure.

![Figure 1: The structure of the Sun](image)

The centre of the Sun is formed by the core, which is considered to extend to about 0.23 solar radii. It is characterized by a very high density, a very high pressure and very high temperatures of about 15,000,000 K. The core is the location where heat is produced by fusion processes. The rest of the Sun is heated by this energy that is transferred outward and that leaves the Sun finally as electromagnetic radiation or kinetic energy of particles.

The adjacent layer is the radiative zone, from about 0.23 to about 0.7 solar radii. Its mean temperature is about 7,000,000 K. Density and pressure are much lower than in the core too. The energy generated in the core is transported through the radiative zone by radiative processes experiencing successive absorption and reradiation.

In the convective zone, which follows the radiative zone (from 0.7 solar radii to the full solar radius), the solar plasma has a much lower density (about 0.15 g/m$^3$) and also a lower temperature (about 2,000,000 K) than in the radiative zone. The energy transport occurs through convection.

The visible surface of the Sun is the photosphere. It is the layer from where the visible solar radiation with its continuous spectrum is emitted. Above the photosphere, electromagnetic radiation is free to

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These mean values vary considerably within the different layers. Density and pressure decrease continuously from the core to the outer parts. Temperature decreases continuously up to the photosphere and chromosphere and increases towards the corona.
propagate into space. The photosphere is quite a thin layer of between tens and hundreds of kilometres with a very low density. Its temperature is about 5800 K. The relatively thin (around 10,000 km) chromosphere and the very extensive (up to 20 solar radii) corona are visible only during a solar eclipse and represent the Sun atmosphere. They are nearly transparent, although the name of the chromosphere originates from its slightly reddish colour.

1.2 Nuclear fusion in the Sun

As mentioned above, the energy in the Sun is generated by nuclear fusion processes. In these processes, hydrogen is converted into helium. Currently, the Sun consists of about 75% hydrogen (weight percentage), 23% helium and 2% other elements. The most important reaction chain is the so-called proton-proton chain. Another reaction chain, which is much less frequent than the former one, is the carbon-nitrogen-oxygen chain. The summarized reaction equation (not taking into account the intermediate steps of the reaction chains) in both cases is the following:

\[ 4 \text{p}^+ \rightarrow ^4\text{He}^{2+} + 2 \text{e}^+ + 2 \nu + \Delta m c^2 \]

Protons (hydrogen nuclei, \(\text{p}^+\)) fuse into helium (\(^4\text{He}^+\)), releasing additionally positrons (\(\text{e}^+\)) and neutrinos (\(\nu\)). This process results in a mass defect and, consequently, energy is released, 26.7 MeV per reaction. The total solar mass defect per second amounts to about \(4.3 \cdot 10^9\) kg. According to the mass-energy equivalence \(E = mc^2\), this corresponds to a radiation emission power of the Sun of about \(3.85 \cdot 10^{26}\) W. The following figure shows once more the mentioned processes and numbers.

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7 As the Sun is a gaseous body it seems a bit arbitrary to speak of a solar atmosphere. However, it has become common to call the layers above the photosphere, i.e. the layers above the visible layer of the Sun, the solar atmosphere.
1.3 The Sun as a thermal radiator

In the following paragraphs, some concepts of radiation physics will be applied, which bear a close relation to each other and which can be mistaken easily: radiation, irradiation, radiant energy, radiant flux, irradiance, and radiant emittance. So, let us start with the following terminological and conceptual clarification:

**Radiation:** “radiation” is used here in a very general way, neither indicating a specific physical quantity nor having a specific dimension. Radiation is a transport process, in which energy propagates through a medium or through empty space. In general, in this chapter electromagnetic radiation is considered.

**Radiant energy:** energy of electromagnetic waves (unit: joule [J])

**Radiant power:** radiant energy per unit time (unit: watt [W], joule per second [J/s])

**Irradiation:** This terminus is used generally for the process in which an object is exposed to radiation. As a quantity it is used for the incident radiant energy per unit area (unit: joule per square metre [J/m²])

**Irradiance:** incident radiant power per unit area (unit: watt per square metre [W/m²])

**Radiant emittance:** emerging radiant power per unit area of emitting surface (unit: watt per square metre [W/m²])

The energy produced in the fusion processes in the Sun core leaves the Sun as radiative energy. A part of this energy is released as radiation of matter (solar wind) and the bigger part as electromagnetic radiation. The latter is the radiation that is important in our context, because it is the source for the operation of CSP systems.
The electromagnetic radiation that is emitted by the Sun resembles in its spectrum the thermal radiation of a black body at a certain temperature. Therefore, and in order to understand the characteristics of the energy source for CSP systems, it is important to understand thermal radiation. That’s why we will now recapitulate some fundamental radiation physics.

### 1.3.1 Thermal radiation

All physical objects emit electromagnetic radiation. Light is electromagnetic radiation in a certain range of frequencies and wavelengths. Nevertheless, not all bodies emit light. Actually, most visible objects we do not perceive because of the radiation they emit but because of the light they reflect and that originates from other sources. We know this because at night we cannot see the majority of the objects we can see in daylight.

Nevertheless, there is a way to make objects visible even at night without illuminating them: When we heat them up sufficiently, they will begin to emit light. At first they will glow in dim reddish light. When we continue to heat them up, they will glow more intensely and the colour of the emitted light will change towards yellow and later white. The experience that we can make objects glow by heating them up is a hint on the dependency of the emitted electromagnetic radiation on temperature. Indeed, the emission behaviour of a body depends on its temperature as well as on the object’s material and surface properties. The radiation which is emitted by bodies due to their temperature is called thermal radiation.

Now, there is a common useful idealization in radiation physics. Imagine a body that neither reflects any incident light nor lets it pass through, i.e., it absorbs all the incident electromagnetic radiation. Such a body is called a black body or an ideal radiator. Black bodies do not exist in reality. They are just useful theoretic constructions. In radiation physics, thermal radiation is described at first for ideal black bodies, and the radiation behaviour of real bodies is derived subsequently from the black-body radiation.

At the beginning of the twentieth century, Max Planck formulated his famous law of black-body radiation which describes the spectral composition of the radiation of a black body in dependency of its temperature. One formulation of this law, which indicates the temperature dependent hemispherical emittance per unit surface area and per unit wave length at the wavelength $\lambda$, is this:

$$M(\lambda, T) = 2\pi h c^2 \frac{1}{\lambda^5} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1},$$

(1)

where $h = 6.626 \cdot 10^{-34}$ Js is the Planck constant, $k = 1.38 \cdot 10^{-23} \frac{J}{K}$ the Boltzmann constant and $c = 2.998 \cdot 10^8 \frac{m}{s}$ the speed of light. The spectral composition of the electromagnetic radiation emitted by black bodies at different temperatures which is expressed in this formula is represented in the following figure.
It can be seen that thermal radiation has a continuous spectrum. There are other types of spectra that do not show the same continuous structure: emission spectra and absorption spectra. Emission spectra are produced e.g. by hot gas of low density and consist of characteristic lines, which depend on the type of gas. Absorption spectra are produced when light propagates through partially transparent matter. They have the form of continuous spectra with dark absorption lines. Furthermore, we can observe the following relations between temperature and radiation: First, for each temperature there is a maximum of radiation intensity at a certain wavelength. At the left and at the right side of the maximum the spectral radiant emittance $M$ declines continuously. We see also that the wavelength for which the maximum appears varies with temperature. The higher the temperature the smaller is the wavelength of the radiation at the maximum. The corresponding quantitative relation is known as *Wien’s displacement law*. It states that the wavelength at the point of maximum spectral radiant emittance $M$ is inversely proportional to the temperature:

$$\lambda_{\text{max}} = \frac{b}{T}$$  \hspace{1cm} (2)

where $b = 2.8978 \cdot 10^{-3}$mK. This relation permits to derive the temperature of a body from the spectrum of the radiation it emits. The discontinuous black line in the following figure indicates the location of the power maximum for different temperatures.
Second, the total radiant emittance, represented by the surface below the different curves (see figure 2.5), varies also with temperature. Higher temperature is correlated with higher radiant emittance. This relation is quantitatively specified in the Stefan-Boltzmann law. It states that the total power radiated by a black body is directly proportional to the fourth power of the body’s absolute temperature:

$$P = A \cdot \sigma \cdot T^4$$

where \( \sigma = \frac{2\pi^5 k^4}{15c^2 h^3} = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4} \) is the Stefan-Boltzmann constant and \( A \) the body surface area. These three interconnected laws, Planck’s black-body radiation law, Wien’s displacement law and the Stefan-Boltzmann law represent an important physical background in order to understand thermal radiation and solar radiation in particular.
Wien’s displacement law can be mathematically derived by the differentiation of Planck’s formula with respect to the wavelength and the Stefan-Boltzmann law can be derived by the integration over all wavelengths.

### 1.3.2 The Sun spectrum

Black bodies are an idealization. There are no black bodies in reality. Neither the Sun is one. Nevertheless, radiation spectrum and radiation power of the Sun resemble the spectrum and the intensity of the radiation of a black body at around 5780 K.
Figure 6 shows the spectral composition of the solar radiation in comparison to the spectrum of a black radiator at 5777 K. This temperature is chosen because the radiant power of the Sun is equal to the radiant power it would have if it was a black radiator at 5777 K. In figure 6, the area below the two curves is the same. Accordingly, often an effective Sun temperature of 5777 K is indicated.

However, the two curves are not identical, because the Sun is not exactly a black radiator. First, the Sun is not a black body and, second, as we have seen, the Sun does not have one uniform temperature, but it consists of different layers with different temperatures. Furthermore, the solar spectrum is interrupted by dark lines due to absorption processes in the Sun atmosphere.

Taking Wien’s displacement law as the criterion, the effective Sun temperature would be higher (about 6300 K), because the observed radiation maximum in the Sun spectrum is at a shorter wavelength than it should be if the Sun was a black radiator at 5777 K.

The biggest part of the solar spectrum is in the range of visible light (the lower wavelength limit is taken to be between 360 and 400nm and the upper limit between 760 and 830nm). Smaller parts are in the ultraviolet and in the infrared range. Radiation with a wavelength below about 400nm is ultraviolet radiation (which sometimes divided into three sub-ranges: UV-A: 315-400nm, UV-B: 280-315nm, UV-C: 100-280nm). Radiation with a wavelength above 800nm is infrared radiation.
2 Solar radiation on the Earth’s surface

2.1 Solar constant

We want to know more about the solar radiation at the Earth’s surface, which is the direct energy source for the operation of CSP plants. In a first step we will determine the power of solar radiation per unit area at the outer border of the Earth's atmosphere. It is nearly constant and the value is called the solar constant. Note well, we are not speaking about the irradiance on the Earth’s surface, but about the radiant power outside the terrestrial atmosphere. The influence of the atmosphere on the radiation until it reaches the Earth’s surface will be the subject of subsequent sections.

The solar constant depends basically on three parameters: the temperature of the Sun, more precisely of the photosphere which emits the major part of the radiation that leaves the Sun, the size of the Sun, and the distance between Sun and Earth. We just mentioned that the temperature of the Sun surface can be considered to be 5777 K. Additionally, we know that the Sun radius $r_S$ is about $6.965 \times 10^8$ m and that the mean Sun-Earth distance $r_{SE}$ amounts to approximately $1.496 \times 10^{11}$ m. On the basis of these three parameters and considering the Sun as a black body (and thus simplifying the complex reality, yet we know that it is only approximately a black body), we can calculate the solar radiation power arriving at the Earth.

First, the Stefan-Boltzmann law permits to calculate the total solar radiation power:

$$P_S = \sigma T^4 \cdot 4\pi r_S^2$$

$$= 5.67 \cdot 10^{-8} \cdot 5777^4 \cdot 4\pi \cdot (6.965 \cdot 10^8)^2 \text{ W}$$

$$= 3.85 \cdot 10^{26} \text{ W}$$

With this power, the Sun emits radiation into the space, as mentioned before. Now, the same total radiation power arrives at any sphere around the Sun. No radiation gets lost on its way. So, if we consider the sphere around the Sun on which the Earth’s orbit is located then we know that the same radiant energy that leaves the Sun arrives at this sphere. That means that the irradiance at this sphere, i.e. the solar constant $G_{SC}$ we are looking for, can be calculated taking the total solar radiation power $P_S$ and dividing it by the area of the considered sphere with the radius $r_{SE}$:

$$G_{SC} = \frac{P_S}{4\pi r_{SE}^2} = \frac{3.85 \cdot 10^{26} \text{ W}}{4\pi (1.496 \cdot 10^{11} \text{ m})^2} \approx 1367 \frac{\text{W}}{\text{m}^2}$$

---

8 The indicated value is known as the Astronomical Unit (AU). It represents roughly the mean Sun-Earth distance.

9 Actually, it is not necessary to calculate at first the total solar radiation in order to reach the indicated result. There is a shorter way: The power of the emitted solar radiation per unit area is reciprocally proportional to the square of the distance from the centre of the Sun. (Consider spheres around the Sun’s centre and take into account that the same radiation is distributed on spherical areas which are directly proportional to the square of their radius.) So, assuming that the radiation power per square meter on the Sun’s surface is $\sigma T^4$ and taking into account that the surface has a distance from the Sun’s centre of $r_S$, we get the same result more directly: $G_{SC} = \sigma T^4 \cdot r_S^2 / r_{SE}^2$. 
Figure 7: Determination of the Solar constant

Remember that this is the radiant power per square meter incident on a surface on top of the atmosphere and in normal direction to the incident rays. The irradiance on the Earth’s surface will be different; in no case it will reach this value. We neither should expect that the calculated value will be exactly the real value of the solar constant; the very idealization of the Sun as a black body should imply an error. Moreover, the mere supposition that there is one constant value cannot be true. One reason is that the Sun-Earth distance is not constant. There are regular variations during the year and further long-term variations. So, the solar constant can be understood at best as an average value over a certain time period.

Several experiments were made to measure the solar constant. High altitude aircrafts, balloons, and satellites permitted direct measurements of solar radiation outside most or all of the Earth’s atmosphere. In 1982, as a result of the different measurements, the World Meteorological Organization fixed the average value of 1367 $\frac{W}{m^2}$ as the solar constant.\textsuperscript{10}

There are certain processes which cause variations in the power of the solar radiation that arrives at the atmosphere. The main short period variation of the solar radiation at the outer border of the atmosphere results from the fact that the Earth’s orbit around the Sun is not exactly a circle, but an ellipse. That means that sometimes the Earth is closer to the Sun than at other times. However, the eccentricity of the elliptic orbit is quite small. The Earth-Sun distance increases and decreases during a year by about $\pm 1.7\%$ in relation to the mean distance.\textsuperscript{11} As a consequence, the solar irradiance on top of the Earth’s atmosphere varies by about $\pm 3.3\%$ relative to the indicated mean value of 1367 $\frac{W}{m^2}$. The maximum irradiance (i.e. the minimum Earth-Sun distance) is reached in January and the minimum (i.e. the maximum Earth-Sun distance) is reached in July. Taking into account the eccentricity effect with a variation of $\pm 3.3\%$ of the irradiance in relation to the mean value, the solar irradiance outside

\textsuperscript{10}See Wagemann/Eschrich 1994, 8.

\textsuperscript{11}As indicated above, the mean distance is approximately $1.496 \cdot 10^{11}$m. The maximum distance is about $1.521 \cdot 10^{11}$m (around July 5\textsuperscript{th}) and the minimum distance is about $1.471 \cdot 10^{11}$m (around January 3\textsuperscript{rd}).
the atmosphere for a given day of the year (DoY, with DoY = 1 on January 1) can be approximated in the following way\textsuperscript{12}:

\[
G_{on} = G_{SC} \left( 1 + 0.033 \cos \frac{360 \circ \text{DoY}}{365} \right)
\]  
(4)

There are further short time irregularities due to changing sunspot activity, but these irregularities are very small compared to the eccentricity effect. On the other hand, there are long term variations due to the Milankovitch cycles which consist in variations in eccentricity, axial tilt and precession of the Earth’s orbit. Yet, these variations are so slow that they do not have any practical importance at a human time scale. Finally, at huge time dimensions, there is the astronomically long time variation of the Sun activity according to the live cycle of a star. However, all the latter variations are insignificant for practical purposes. Even the eccentricity effect can be disregarded in many practical contexts. For many practical uses it is sufficient to suppose that there is a constant radiant power arriving at the terrestrial atmosphere the value of which is determined by the solar constant. In addition, concerning the solar irradiance at the terrestrial surface, the variation of the solar radiation outside the terrestrial atmosphere is small compared to the more important variations that result from passing through the atmosphere.

The total radiant power \( \dot{Q}_E \) the Earth receives from the Sun can be determined multiplying the solar constant by the cross sectional area of the Earth. Taking \( r_E = 6371 \text{km} \) as the mean Earth radius we get:

\[
\dot{Q}_E = \pi r_E^2 G_{sc} = \pi \cdot (6371 \cdot 10^3 \text{m})^2 \cdot 1367 \text{Wm}^{-2} = 1.74 \cdot 10^{17} \text{W}
\]

From this value we can calculate the total solar energy \( Q_E \) received on Earth during one year (= 8760h):

\[
Q_E = 1.74 \cdot 10^{17} \text{W} \cdot 8760 \text{h} = 1.52 \cdot 10^{18} \text{kWh}
\]

\textsuperscript{12} Spencer (1971) indicates a more exact formula:

\[
G_{on} = G_{SC}(1.00011 + 0.034221 \cos d + 0.00128 \sin d + 0.000719 \cos 2d + 0.000077 \sin 2d) \text{ where } d = 2 \pi (\text{DoY} - 1)/365.
\]
The dimension of this energy becomes impressive if we compare it to the total world primary energy supply, which amounted to $1.4 \cdot 10^{14}$ kWh in 2007.\textsuperscript{13} This means that the Earth receives about 10,800 times more solar energy than the humanity needs actually for its primary energy consumption.

Furthermore, the calculated radiant energy yields a mean irradiance at the Earth of
\[
\overline{G}_E = \frac{P_E}{4\pi r_E^2} = 341.75 \text{ W/m}^2 \text{.}^\text{14}
\]

However, remind, once more, that we speak about the radiation outside the atmosphere. The irradiance at the Earth’s surface is much smaller because of scattering, reflection and absorption and reemission processes in the atmosphere. Additionally, it is modified in its spectral composition.

### 2.2 Radiation extinction processes in the atmosphere

Several radiation attenuating effects occur when radiation crosses the atmosphere. Generally, they are called extinction processes. There are two general classes of extinction processes: absorption and scattering (being reflection a special case of scattering).

Absorption means that the energy of a photon is taken up by matter. Scattering means that radiation is deviated from straight propagation.

- Atmospheric absorption is a process of radiation extinction which reduces the available solar radiation at the Earth’s surface considerably. Some constituents of the atmosphere absorb radiation of a certain spectral range. Ozone ($O_3$) in the upper atmosphere absorbs almost completely short-wave radiation at wavelengths below 290nm. Above 290nm ozone absorption decreases, until at 350nm there is nearly no absorption. Another weak ozone absorption band is near 600nm.

- Water vapour absorbs strongly in the infrared part of the solar spectrum, with absorption bands at 1, 1.4 and 1.8μm. Carbon dioxide is another strong absorber of infrared radiation. Due to both gases, $H_2O$ and $CO_2$, the radiation transmission through the atmosphere is very low at wavelengths above 2.5μm.

- Finally, oxygen and nitrogen absorb radiation over a large wavelength range.

- Scattering is a process in which radiation is forced to deviate from a straight trajectory by non-uniformities in its way (molecules, dust particles etc.). In the case of solar radiation, two types of

\textsuperscript{13} International Energy Agency (IEA), Key World Energy Statistics 2009

\textsuperscript{14} Note that this value includes also nighttime.
scattering are distinguished: Rayleigh-scattering and Mie-scattering. Which of these two types of scattering happens depends especially on the size of the non-uniformities.\textsuperscript{15} Because of scattering processes, solar radiation reaches the Earth’s surface partially as diffuse radiation. Not all radiation arrives as direct (or beam) radiation.

**Rayleigh-scattering**, named after the English physicist John W. Strutt Rayleigh (1842-1919), is the scattering of electromagnetic radiation by particles which are much smaller than the wavelength of the radiation. In the case of light, with a wavelength between 380nm and 780nm, these particles are individual atoms or molecules. Rayleigh-scattering follows a $\lambda^{-4}$-law. That means that light with a smaller wavelength will be scattered much more than light with a longer wavelength. The sky is blue because the blue light, with its shorter wavelength, is scattered much more than the light of other spectral ranges. Additionally, the Sun looks reddish at sunrise and at sunset because the longer way of the radiation through the atmosphere provokes that a bigger fraction of light with shorter wavelength will be scattered away, so that the beam radiation contains a higher fraction of light with longer wavelengths, e.g. red light. The direction of the scattered light follows a $(1 + \cos^2 \alpha)$-pattern, where $\alpha$ is the angle between the beam direction and the angle after the scattering.

**Mie-scattering**, named after the German physicist Gustav Mie (1868-1957), is the scattering of electromagnetic radiation by particles whose diameter is of about the same dimension as the wavelength or larger. In the atmosphere, water droplets, ice crystals and aerosol particles cause Mie-scattering. Mie-scattering does not have a similarly clear-cut $\lambda$-dependency as Rayleigh-scattering; it is less wavelength-selective.\textsuperscript{16} Additionally, while Rayleigh-scattering is quite exclusively a function of Air Mass, Mie-scattering depends strongly on local conditions, especially on air pollution and cloudiness. The white appearance of clouds, for instance, is an effect of Mie scattering at water droplets.

Concerning the scattering direction, Mie-Scattering has a strong forward pattern.

The following table shows once more the principal characteristics of the two kinds of scattering:

<table>
<thead>
<tr>
<th>Table 1: Comparison of Rayleigh-Scattering and Mie-Scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rayleigh-Scattering</strong></td>
</tr>
<tr>
<td>particle size $\ll$ wave length (air molecules)</td>
</tr>
<tr>
<td>strong dependence on wave length: $-\lambda^4$</td>
</tr>
<tr>
<td>(wave length selective process ⇒ blue colour of the sky)</td>
</tr>
<tr>
<td>general scattering form:</td>
</tr>
</tbody>
</table>

Scattering does not convert radiation into other forms of energy. Nevertheless, it reduces beam radiation. If we consider CSP systems, which use only beam radiation, then scattering involves a loss of useable radiation. Additionally, the incoming radiation is partly scattered back to space and does

\textsuperscript{15} The size is not the only criterion. The shape of the non-uniformities is also important.

\textsuperscript{16} Mie-scattering can be described by the formula $\tau = e^{-m\beta\lambda^{2m}}$ (Eicker 2001, p. 37), where $\tau$ is radiation transmission, $m$ Air Mass, $\beta$ a parameter that varies between 0 for very clear sky and 0.5 for very turbid sky and $\alpha$ a function of the size distribution of the aerosol particles that usually is between 0.25 and 2.5 with an average of 1.3 $\pm$ 0.5.
not reach the terrestrial surface, so that there is a real reduction of total irradiance on the Earth’s surface. About one fifth of the total radiation is reflected back to space.\textsuperscript{17}

Clouds have the most important and, as we know from our experience, the most changing effects on the amount of solar radiation that reaches the terrestrial surface, due to the mentioned processes of reflection, absorption and scattering.

Figure 10 and 11 show the radiation reduction effects of the different extinction processes. As to be seen in 11, the radiation reduction in relation to the extraterrestrial radiation is very strong in the ultraviolet range and also quite strong in the infrared range. In the visible range, the absolute reduction is high, but in relation to the extraterrestrial radiation the reduction is lower than in the ultraviolet and infrared range.

**Figure 10:** radiation reduction through atmospheric extinction processes I

**Figure 11:** radiation reduction through atmospheric extinction processes II (source: C. Hoyer-Klick: Introduction to Solar Resource Assessment)

\textsuperscript{17} See Heinemann, Energy Meteorology Script p. 29
2.3 Air Mass

The radiation extinction effect of the atmosphere depends on different aspects like aerosol concentration, humidity and especially cloudiness. These conditions are highly variable at a given place and they can be determined only by measurement. There is a further aspect, which is known without any measurement, but just by the geographical location and by time: the optical path way of the direct solar radiation through the atmosphere. The radiation attenuation depends on the latter in the following way: the longer the way through the atmosphere the stronger its radiation attenuation. The path length of the solar radiation from the top of the atmosphere to a given place on the Earth’s surface, in turn, will be a function of the geographic altitude of the place and of the solar zenith angle $\theta_z$, i.e. the angle between the Earth surface normal and the line to the Sun. We can concretise the relationship in the following way: if $\theta_z$ is 0°, i.e. if the Sun is at the zenith, the light beam has to travel the smallest possible distance inside the Earth’s atmosphere until it reaches the surface. In contrast, if the Sun is near the horizon, the path through the atmosphere will be very long. On this basis it is possible to define a relative measure of the atmospheric mass through which beam radiation passes to reach the Earth’s surface.

The light has to travel through the minimum mass of atmosphere if the Sun is in the zenith. This mass of atmosphere receives the value 1, if a place at sea level is considered. All other possible values will be related to this minimum value. For instance, if the zenith angle is 60°, then the path length through the atmosphere will be double, i.e. it receives the value 2.\(^{18}\) This value is called Relative Air Mass, or simply Air Mass (AM). In a first approximation it is calculated according to:

$$AM = \frac{1}{\cos \theta_z} \quad (5)$$

The following figure demonstrates the dependence of the Air Mass on the incidence angle of solar radiation:

Equation (5) can be only an approximation because this formula takes the terrestrial atmosphere as plane-parallel. But, the atmosphere has a curvature which reduces the real Air Mass for zenith angles $\theta_z > 0$. The curvature effect becomes important especially for large zenith angles. For some solar energy applications the indicated approximation is sufficient. For others the curvature effect has to be taken into account.

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\(^{18}\) The atmosphere has different properties at different altitudes. Its density, for instance, decreases at higher levels. And the concentration of certain gases and of aerosols also varies with altitude. Nevertheless, whatsoever are these variations and whatsoever are their specific effects, these effects will be approximately proportional to the path length within the terrestrial atmosphere. So, the application of the simple proportional Air Mass as a measure for the attenuating effect of the atmosphere can be considered as justified.
into account. An Air Mass formula, which is more exact especially for large zenith angles, has been identified empirically with

\[ AM = \frac{1}{\cos \theta_z + 0.51(93.885 - \theta_z)^{-1.253}} \]  

(6)

Not only the zenith angle determines the Air Mass, but also the altitude of a given place. Taking into account the altitude effect, (6) has been completed to:

\[ AM = \frac{\exp(-0.0001184 \cdot h)}{\cos \theta_z + 0.51(93.885 - \theta_z)^{-1.253}} \]  

(7)

where \( h \) is the altitude above sea level.

### 2.4 Direct, diffuse and reflected radiation

The different extinction processes provoke that not all radiation that reaches the Earth atmosphere reaches the ground. Actually, as we will see later, only about 52% hits the Earth’s surface. Additionally, scattering provokes that a part of the radiation, that reaches the ground, gets there as diffuse radiation rather than direct radiation (or beam radiation). Diffuse radiation does not have a preferred direction.

Direct radiation is radiation that arrives at the Earth’s surface in a straight line from the Sun. CSP systems can use direct radiation only. Non-directional radiation cannot be concentrated and, consequently, cannot be used in these systems.

There is yet another fraction (besides the mentioned diffuse radiation) that contributes to the non-directional radiation. This fraction is not owed to scattering but to reflection on the ground: reflected radiation. It depends on the ground reflectivity. As we know, it varies considerably being much higher at fresh snow than at a green meadow.

The irradiance on a surface in the atmosphere or on the ground is always the sum of these three components: irradiance due to direct radiation, diffuse radiation and reflected radiation. This sum is called total or global irradiance \( G = G_b + G_d + G_r \) illustrated in figure 13.

![Figure 13: Irradiance due to direct, diffuse and reflected radiation](image_url)

\(^{19}\) (2.6) and (2.7) were proposed by Kasten (1966) and are still used in the SOLEMI Method in the DLR (see DLR 2008).
As CSP systems can use only direct radiation, in the following we will be concerned much more with direct radiation than with diffuse and reflected radiation.

2.5 Direct radiation: Turbidity

We now want to know how we can determine the energy available from direct radiation at a given time and at a given place. We know that there is no direct radiation when the sky is covered by clouds. That’s why we have to consider only clear sky conditions. However, as we have seen above, we have to account for the mentioned atmospheric extinction processes in order to derive the irradiance on the ground from the irradiance outside the atmosphere. We know that a clear sky may be more or less turbid due to Rayleigh scattering, Mie-scattering and absorption.\(^{20}\) In order to express the turbidity, a transmission factor can be attributed to each of the extinction processes. These transmission factors are called \(\tau_{rs}, \tau_{ms}\) and \(\tau_{ab}\), respectively. They indicate the ratio of the radiant flux transmitted through the atmosphere (considering exclusively the reduction due to the respective extinction process) to the incident radiation at the top of the atmosphere. For the ratio of the irradiance \(G_{bn}\) at the terrestrial surface and on a plane perpendicular to the direct radiation (direct normal irradiance) to the solar constant \(G_{sc}\) we can conclude:

\[
\frac{G_{bn}}{G_{sc}} = \tau_{rs} \tau_{ms} \tau_{ab} \tag{8}
\]

Furthermore, extinction processes in the atmosphere can be described generally by

\[
G_{bn} = G_{sc} e^{-x \cdot AM}, \tag{9}
\]

where \(x\) is the extinction coefficient with the unit \([m^{-1}]\) and AM the relative optical path length (Air Mass). Let’s imagine a so-called Rayleigh-atmosphere, that means an atmosphere that does not show any Mie scattering nor absorption, i.e. \(\tau_{ms} = \tau_{ab} = 1\), and where the only extinction process is Rayleigh scattering. In this case we get the following extinction coefficient:

\[
x_1 = -\frac{\ln \tau_{rs}}{AM}. \tag{10}
\]

If we consider a realistic atmosphere with \(\tau_{ms} < 1\) and \(\tau_{ab} < 1\), we get

\[
x_2 = -\frac{\ln (\tau_{rs} \tau_{ms} \tau_{ab})}{AM} = -\frac{\ln \tau_{rs} - \ln \tau_{ms} - \ln \tau_{ab}}{AM} \tag{11}
\]

The ratio of these two extinction coefficients is called the turbidity factor \(T_L\):\(^{21}\)

\[
T_L = \frac{x_2}{x_1} = \frac{-\ln \tau_{rs} - \ln \tau_{ms} - \ln \tau_{ab}}{-\ln \tau_{rs}} = 1 + \frac{\ln \tau_{ms} + \ln \tau_{ab}}{\ln \tau_{rs}} \tag{12}
\]

Inserting \(x_2 = T_L x_1\) (from the definition of \(T_L\)) in (9) and with (10) we get

\(^{20}\) We do not take into account reflection because reflection is basically bound to clouds, but here we consider only clear sky conditions.

\(^{21}\) It is also called the Linke turbidity factor.
That means that the turbidity factor indicates the extinction in the real atmosphere in relation to the extinction in a Rayleigh-atmosphere. It indicates how many such Raleigh-atmospheres would be equivalent to the real atmosphere (at a given place and time) concerning their extinction capacity.\(^{22}\) The turbidity factor in a pure Rayleigh-atmosphere would be 1. In the real atmosphere it is always greater than 1.

The reason why the turbidity is defined that way is that Rayleigh-scattering in the atmosphere can be considered as constant and independent on locally and temporally changing conditions, while Mie-scattering and absorption vary corresponding to locally and temporally variant circumstances. Figure 14 shows empirically determined turbidity factors over the year for different types of places in Central Europe. It is not surprising, of course, that the lowest values are to be found in the high mountains, where the air is very clean, and the highest in industrial areas, where pollution is higher than in other areas. What can also be seen is a seasonal variation. The reason of this variation are different water vapour contents in summer and in winter. Higher water vapour contents in summer provoke a stronger radiation reduction.\(^{23}\)

Empirically, the general approach \(G_{bn} = G_{sc}e^{-\gamma AM}\) was determined as follows (using in one expression the solar altitude angle \(\alpha_s\) and in the other the zenith angle \(\theta_z\)):

\[
G_{bn} = G_{sc}e\left(-\frac{T_L}{0.9+9.4\sin\alpha_s}\right) = G_{sc}e\left(-\frac{T_L}{0.9+9.4\cos\theta_z}\right)
\]

\(^{13}\)\(\text{(13)}\)

\[G_{bn} = \tau_{rs}^{T_L}G_{sc} .\]

\(^{22}\) See Kleemann/Meliß 1993, 40f.

\(^{23}\) Additionally, a minor variation during the day was registered. For European conditions, a certain increase at midday in summer and a weak decrease at midday in winter were measured (see Gassel 1997, 18).

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\(\text{Figure 14: Turbidity factor during the year at different places in Central Europe (Gassel 1997, 18)}\)

---

\(^{22}\) See Kleemann/Meliß 1993, 40f.

\(^{23}\) Additionally, a minor variation during the day was registered. For European conditions, a certain increase at midday in summer and a weak decrease at midday in winter were measured (see Gassel 1997, 18).
2.6 Direct radiation on tilted planes

Equation (14) indicates the irradiance of direct radiation on planes that are perpendicular to the radiation direction, i.e. it indicates the direct normal irradiance. Let’s consider additionally the irradiance of direct radiation on horizontal planes and, finally on tilted planes. The incidence angle is defined as the angle between the radiation direction and the normal of the irradiated plane. In the case of a plane perpendicular to the radiation direction, the incidence angle is $0^\circ$. Now, imagine a plane that is exposed to a beam under different incidence angles, as to be seen in figure 15. The irradiance, i.e. the irradiated power per square meter, on the illuminated part of the plane is inversely proportional to the area illuminated by the beam. The area that is illuminated depends on the incidence angle $\theta$. If the direct normal irradiance is $G_{bn}$, then the irradiance on a tilted plane $G_{bt}$ under an incidence angle $\theta$ is

$$G_{bt} = \cos \theta \cdot G_{bn} \ .$$

The irradiance reduction of direct radiation due to non-normal orientation of the irradiated surface in relation to the radiation is sometimes called the cosine effect.

In the case of a horizontal plane, the incidence angle is equal to the solar zenith angle. The direct horizontal irradiance $G_{h}$ is, then

$$G_{h} = \cos \theta_{z} \cdot G_{bn} \ .$$

Combining equations (15) and (16) we can express $G_{bt}$ also as a function of $G_{h}$:

$$G_{bt} = \frac{\cos \theta}{\cos \theta_{z}} G_{h}$$

Figure 15: Irradiance on tilted planes

Equation (14) can be used to generalize equation (13) for any tilted plane:
What is still missing is the determination of the incidence angle \( \theta \), to which we will return below.

### 2.7 Diffuse and reflected radiation

As mentioned above, we concentrate here on direct or beam radiation. However, in order to round out the topic of radiation on tilted surfaces in this paragraph, we will add some general remarks and rough formulae concerning diffuse and reflected radiation.

a) **Diffuse radiation** can be characterized by means of a clearness index \( K \). \( K \) is defined as the ratio of global radiation to the extraterrestrial radiation:

\[
K = \frac{G}{G_{sc}}
\]

This clearness index can be determined empirically by measurements. It allows ascertaining the diffuse radiation from the global radiation, using empirically determined correlations between global radiation and diffuse radiation as a function of the actual clearness index. According to Origill/Hollands \(^{25}\), the following correlations hold:

\[
\frac{G_d}{G} = \begin{cases} 
1.0 - 0.249K & \text{for } K < 0.35 \\
1.557 - 1.84K & \text{for } 0.35 \leq K \leq 0.75 \\
0.177 & \text{for } K > 0.75
\end{cases}
\]

We suppose that the diffuse radiation at a given place and time is isotropic, this means, that it is equal in all directions from the half-space above the horizontal plane (non-directed radiation coming from the Earth’s surface, i.e. reflected radiation, is here not subsumed under diffuse radiation). The irradiance on a tilted surface, due exclusively to diffuse radiation, is, then:

\[
G_{dt} = \frac{1 + \cos \beta}{2} G_d ,
\]

where \( \beta \) is the tilt angle as defined above. A surface facing to the zenith (\( \beta = 0^\circ \)) receives the full diffuse radiation, while a surface facing to the ground (\( \beta = 180^\circ \)) receives no diffuse radiation at all.\(^{26}\)

b) **Reflected radiation** depends on the global radiation and on the reflectivity \( R \) of the Earth’s surface, i.e. on its albedo. Albedo is the ratio of diffusely reflected radiation to incident radiation. It varies between 0.1 for dark, wet soil or forest to more than 0.9 for fresh snow. Typical values for many

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\(^{24}\) This clearness index \( K \) is not to be confounded with the clearness index \( k_t \), in the section 5.2, which is the ratio of the global irradiance at a given place and a given time to the global irradiance that would exist under clear sky conditions.

\(^{25}\) See Origill/Hollands 1977.

\(^{26}\) It is quite a big simplification to take diffuse radiation as isotropic. The indicated equation is, thus, only a very rough approximation. There have been elaborated more complex expressions for the irradiance on tilted surfaces that take into account the circumsolar concentration of diffuse radiation and a higher diffuse radiation concentration near the horizon.
places (without conditions responsible for extreme albedo values as, for instance, snow and ice) are between 0.2 and 0.3.

We suppose once more that the reflected irradiance at a given place and time is isotropic, this means, that it is equal in all directions from the half-space below the horizontal plane (the reflected radiation in the sense we apply here does not come from the sky, but from the ground). The irradiance on a tilted surface, exclusively due to reflected radiation, is, then:

\[ G_{rt} = \frac{1-\cos \beta}{2} RG. \]  

(21)

A surface facing the zenith does not receive any reflected radiation, while a surface directly facing the Earth’s surface receives the full reflected radiation.  

2.8 Radiation balance of the Earth

The Earth is in a rough radiation equilibrium in the sense that the radiant power incident on the Earth equals roughly the radiant power leaving the Earth. Taking into consideration that the mean solar irradiance on the Earth is \( \bar{g}_E = 341.75 \text{ W/m}^2 \), and supposing (counterfactually) that the Earth has a uniform surface temperature and that it does not have any atmosphere, the application of the Stefan Boltzmann Law \( \bar{g}_E = \sigma T^4 \) yields the following surface temperature:

\[ T = \frac{4 \bar{g}_E}{\sigma} = 5.5^\circ C. \]

Now, we know that it is not that cold on Earth. The real mean surface temperature is about 15°C. This is the case because of the terrestrial atmosphere and its multiple effects (greenhouse effect). As already mentioned above, only about 50% of the incident solar radiation reaches the Earth’s surface, approximately half of it in form of beam radiation and the other half in form of diffuse radiation. Among the other 50%, which do not reach the terrestrial surface, about 25% are absorbed by ozone, water vapour, aerosol particles, clouds etc. and the rest is backscattered into space.

---

27 This equation supposes that there is no repercussion of the tilted surface itself on the reflection due to shading. Obviously, a sufficiently big surface that is sufficiently closed to the reflecting terrestrial surface does not receive the radiant power indicated in (30), because it obstructs the incoming radiation.

28 It is important to emphasize that the indicated value is valid only for a planet with a uniform surface temperature (which, of course, is impossible). It cannot be understood as a mean temperature of a planet similar to the Earth (i.e. with temperature differences due to day-night-changes, seasonal variations and different radiation angles at different latitudes) but without atmosphere. The to-the-power-of-four-dependency of the radiation emission on temperature does not allow interpreting the value as an arithmetic mean at a planet with non-uniform temperature. Indeed, a known mean radiant emittance of a body does not allow determining any mean temperature of the body.

29 This value is not to be confounded with another theoretical terrestrial surface temperature, which is often indicated in the literature: -18°C. This is the temperature of an Earth-like planet with homogenous temperature disregarding the greenhouse effect, but including other atmospheric effects (in particular, reflection of incident radiation back into the space).
Figure 16: short-wave radiation balance

About 30% of the radiation that reaches the Earth with its atmosphere is reflected or backscattered to space, partially from the clouds, partially from the atmosphere itself, partially from the Earth’s surface (mean planetary albedo ≈ 0.3). The rest is absorbed by atmospheric components, clouds and the terrestrial surface. According to their lower temperature, the absorbed energy is finally emitted in a longer wavelength range. According to Wien’s displacement Law (see above), the radiation emitted from the terrestrial surface (with an average temperature of 288K) lies basically in the mid and long infrared range (with its maximum at about $\lambda = 10\mu m$). Within the system Earth-atmosphere, several radiation and heat exchange processes take place. Just to mention some important of them: A part of the radiation that is emitted by the terrestrial surface is absorbed by the clouds and by other atmospheric components and reemitted, partially into space and partially back to the terrestrial surface so that the atmospheric infrared radiation, i.e. the atmospheric counterradiation, constitutes an important part of the incident radiation on the terrestrial surface. Furthermore, convective processes (wind), latent heat release and absorption in the hydrological cycle and innumerable energy conversion processes, for instance in the biosphere and in the anthroposphere, at the end of which always heat will be released at a temperature near the ambient conditions, complete the complex energy conversion processes.
At the end, all incoming solar radiation that is not reflected to space, i.e. about 70%, will be converted into heat. Approximately 40% are converted directly into heat; about 22% are absorbed for the hydrological cycle (0.003% of which are captured in the flow energy of rivers). 2.5% feed wind, waves and ocean streams, and 0.1% are absorbed for biomass production.

The radiation equilibrium is only a rough equilibrium. Fossil materials like our fossil fuels mineral oil, coal and natural gas exist because solar energy was stored a long time ago in the lithosphere. There was a small surplus of incident solar radiation during the millions of years, when the organic material was accumulated and transformed into fossil materials. Additionally, global climate change processes, particularly global warming and cooling, indicate a temporal disequilibrium between the radiation that reaches and radiation that leaves the Earth. These processes, i.e. the formation of fossil materials and the change of climatic conditions, however, are slow and are generated because of radiation disequilibria that are minuscule if expressed in percentaged differences between incoming and leaving radiation.
3 Geometrical aspects of direct solar radiation

CSP systems use direct solar radiation. They can use only direct radiation because it has a defined direction, and the latter is necessary for radiation concentration. In order to be able to concentrate solar radiation it is necessary to know the direction of the beam radiation or, what is the same, the (apparent) position of the Sun in relation to a terrestrial observer. This section is dedicated to the determination of the direction of the beam radiation.

A localization of the Sun respective a terrestrial observer depends on the following aspects: the position of the observer on the Earth, the general Sun-Earth geometry, refraction processes in the atmosphere and, finally, time.

An exact calculation of the apparent Sun position is very complex. Several algorithms have been proposed that allow the determination of the direction of the beam radiation at different accuracy levels. Very exact algorithms as, for example, the Michalsky algorithm or the NREL algorithm take into account not only the Sun-Earth geometry, but also radiation refraction in the atmosphere. In the present section we will consider only a rougher approximation. We do not consider refraction and we characterize the observer position in the longitude-latitude coordinate system only, not taking into account the altitude, which would also be relevant for a more exact calculation.

CSP applications need a very accurate determination of the apparent Sun position. The calculations we present in the following are not sufficiently exact, but they represent very good approximations and they are a basis for any elaborated algorithm. Moreover, their understanding provides an important prerequisite for the understanding of solar engineering.

3.1. Sun-Earth geometry

For a description of the general Sun-Earth geometry it is sufficient to consider the changing angular relations between the star and the planet. Therefore we can use a heliocentric or a geocentric perspective.

We begin with the heliocentric perspective. The following figure shows some important relations.
The Earth revolves once a year around the Sun and approximately once a day rotates once around its own axis. The plane in which the Earth revolves around the Sun is called the ecliptic plane. The equatorial plane, i.e. the plane perpendicular to the Earth’s axis that includes the Earth equator, is inclined to the ecliptic plane by approximately 23.5° and the Earth’s axis is also inclined by 23.5° to the normal of the ecliptic plane. This inclination provokes different annually periodic variations in the irradiation conditions at the two hemispheres, which is the cause of the existence of seasons. The Sun is in the equatorial plane at spring equinox and at autumn equinox. At these two points, irradiation conditions on the northern and on the southern hemisphere are equal. Northern summer solstice is the point, where irradiation is maximal on the northern hemisphere and minimal on the southern hemisphere. Northern winter solstice is the point, where irradiation is maximal on the southern hemisphere and minimal on the northern hemisphere.

The Earth’s orbit is not a circle but an ellipse, although with a very small excentricity. The Sun is situated in one of the two focuses. The perihel (closest point on the Earth’s orbit to the Sun) is reached at the beginning of January and the aphel (farthest point on the Earth’s orbit to the Sun) is reached at the beginning of July.
We come now to the geocentric point of view.

![Figure 19: Sun-Earth geometry from the geocentric point of view](image)

This perspective shows an apparent revolution of the Sun around the Earth. The exact form of this apparent revolution depends on the observer’s position on the Earth, more exactly, on the latitude of his standpoint. In figure 19 a standpoint at around 60° northern latitude was chosen. The apparent movement of the Sun is realized in the equatorial plane (at spring equinox and at autumn equinox) or in planes that are parallel to the equatorial planes. It is useful to project the apparent Sun orbit and other curves and points of interest on a virtual celestial sphere, as to be seen in figure 19.

The equatorial plane was explained already for the heliocentric perspective. The horizontal plane is the plane that includes the horizontal line of the observer. It divides the sky into two hemispheres, the upper hemisphere that is visible from the observer’s position and the lower hemisphere that is not visible to him (because the Earth is in the way). The angle between the horizontal and the equatorial plane is $90° - \Phi$, where $\Phi$ is the latitude of the observer’s position.

The zenith is the point on the celestial sphere that is vertically above the observer. The nadir is the point on the celestial sphere that is opposite to the zenith. The north celestial pole is the point where the northern extension of the Earth’s axis subtends the celestial sphere. The opposite point is the south celestial pole. The meridian is the circle on the celestial sphere that is perpendicular both to the horizontal plane and to the equatorial plane. Zenith, nadir, north celestial pole and south celestial pole are located on the meridian.

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30 Of course, this is only roughly true. More exactly, the movement has the form of a very narrow spiral between the indicated curves at summer solstice and at winter solstice.
3.2 Coordinate systems

In order to describe the Sun’s position (and at the same time the direction of direct solar radiation) we need appropriate coordinate systems. As we are not interested in the Earth-Sun distance but only in the direction of the Sun in relation to the Earth, or in relation to some place on the Earth, two angles are necessary and sufficient to determine the Sun’s position. There are two common coordinate systems for mapping celestial objects in relation to the Earth or to some place on Earth, a) the equatorial coordinate system and b) the horizontal coordinate system.

a) The *equatorial coordinate system* takes the equatorial plane, i.e. the plane through the terrestrial equator as the fundamental plane. The two coordinates are the *declination* $\delta$ and the *hour angle* $\omega$. The declination is the angle between the equatorial plane and the line to the Sun. The hour angle is the angular displacement of the Sun east or west of the local meridian due to the rotation of the Earth around its axis.

![Equatorial coordinate system](image)

**Figure 20**: Equatorial coordinate system

The declination is a function exclusively of time (and not, for instance, of geographical position). Corresponding to the inclination of the Earth axis to the orbit of about 23.45° it oscillates with the period of one year between 23.45° and -23.45°. Taking into account that the variation during one day generally is insignificant for practical purposes, the declination can be approximated as a function of the day of the year:\footnote{Cooper (1969). Spencer (1971) gives the more accurate formula: $\delta = (0.006918 - 0.399912\cos d + 0.070257\sin d - 0.006758\cos 2d + 0.000907\sin 2d - 0.002679\cos 3d + 0.001467\sin 3d) \times 180\pi$, where $d = 360° \cdot (DoY - 1)/365$.}

$$\delta = 23.45° \sin \left(360° \frac{284 + DoY}{365}\right) \quad (22)$$
Note that the declination is positive during spring and summer of the northern hemisphere and negative during spring and summer of the southern hemisphere.

The hour angle is also exclusively a function of time, more precisely of the hour of day (HoD). Taking solar time, which means the time based on the apparent angular motion of the Sun across the sky, with solar noon at the moment when the Sun crosses the meridian at the observer’s position, and taking into consideration that the Earth rotates at a rate of 15° per hour, the following relationship between hour angle and time can be formulated:

\[
\frac{\Delta \omega}{\Delta t} = \frac{15^\circ}{h}
\]  

(23)

Defining \( \omega \) (12:00h solar time) = 0° (being positive at morning and negative at afternoon), the hour angle can be determined for any hour.

b) The horizontal coordinate system uses the observer’s local horizontal plane as the fundamental plane.

![Figure 21: Horizontal coordinate system](image)

The two coordinates belonging to the horizontal coordinate system are the solar altitude angle \( \alpha_s \), which is the angle between the horizontal and the line to the Sun, and the solar azimuth angle \( \gamma_s \), which indicates the angular displacement from south of the projection of beam radiation on the horizontal plane. Displacements to the east are negative and to the west positive. Instead of the solar altitude angle we could also use the zenith angle \( \theta_z = 90^\circ - \alpha_s \), which indicates the angle between the vertical (pointing above the observer’s position) and the line to the Sun.

The altitude angle and the azimuth angle for a site with the geographic latitude \( \varphi \) can be calculated on the basis of the equatorial system according to:

\[
\sin \alpha_s = \sin \Phi \sin \delta + \cos \Phi \cos \delta \cos \omega \quad (24/1)
\]

\[
\cos \gamma_s \cos \alpha_s = \sin \Phi \cos \delta \cos \omega - \cos \Phi \sin \delta \quad (24/2)
\]

\[
\sin \gamma_s \cos \alpha_s = \cos \delta \sin \omega \quad (24/3)
\]
In the case that the zenith angle $\theta_z$ is used instead of the solar altitude angle $\alpha_s$, $\sin \alpha_s$ has to be changed for $\cos \theta_z$ and $\cos \alpha_s$ has to be changed for $\sin \theta_z$.

The horizontal coordinate system has the advantage to be more descriptive, because, from the normal human standpoint, we observe the Sun in a horizontal system. Nevertheless, it is less convenient for calculations.

### 3.3 Solar time

The position of the Sun in relation to an observer on Earth is a function of the geographical position of the observer and of time. Time entered the indicated coordinate formulas in two different forms, first, in a rather coarse and unproblematic form as *day of the year* and second, in a much more precise form, as the *solar time*. The day of the year appeared in the formula for the declination $\delta$ and solar time appeared in the formula for the hour angle. In the case of declination it is not necessary to be more precise because declination varies by $46.9^\circ$ just once in half a year. However, the hour angle varies by approximately the same amount in only three hours, as expressed by the indicated formula. So, accuracy in time determination is crucial in this case.

If we used standard time, then the indicated formula $\Delta \omega/\Delta t = (15^\circ)/h$, with $\omega=0^\circ$ at 12:00h would be so inexact that it possibly could be useful for a hiker’s orientation, but not for more technical purposes. Now, there are two options: Either we use standard time and change the simple formula for a much more complex one or we maintain the simple formula and decide to substitute standard time for another kind of timescale. The common decision is to use the short formula for the hour angle and to change from standard time to *solar time*. Solar time is defined in such a way that the Sun passes the meridian always exactly at noon. That means, at 12:00h solar time the Sun is exactly in the south (in the northern hemisphere) or in the north (in the southern hemisphere). Additionally, the angular movement of the Sun satisfies equation (23). A solar day, i.e. the time lapse between two subsequent crossings of the Sun path with the local meridian, has exactly 24 hours at the solar time scale (although it is slightly variable at the standard time scale).

There is a general divergence between standard time and solar time due to two different kinds of reasons.

First, there may be a difference between the observer’s geographical longitude and the reference longitude of his time zone. Remember that a time zone comprises a certain range of longitudes and that there can be at most one reference longitude within this range where the Sun passes the meridian at noon. The Central European Time zone, which reaches from Spain to Poland, for instance, includes an area of more than 33 degrees, where the same standard time applies (see figure 22). However, in western Spain the Sun passes the meridian more than two hours later than in eastern Poland.
Second, at the same longitude the solar noon time, i.e. the time when the Sun passes the meridian, varies over the year (if measured in standard time). That means that the days (in the sense of the time span between two Sun crossings of the meridian) do not have the same length (measured in standard time). There are two reasons for that: a) the elliptical orbit of the Earth and, b) the tilt of the ecliptic plane with respect to the equatorial plane.

a) The Sun performs its apparent rotation around the Earth not only because of the revolution of the Earth, but also because of the rotation of the Earth around the Sun. A planet that rotates around a sun moves with a certain angular velocity around it, and this angular variance causes different illumination situations. But, if the angular velocity changes, then this effect varies too. As the terrestrial orbit is an ellipse, and not a circle, the angular velocity is not constant. This is implied by the second of Kepler’s laws of planetary motion. According to this law, the line joining a planet and the sun sweeps out equal areas during equal intervals of time. As illustrated in figure 23, this involves that the angular velocity is not constant. Consequently, the length of days, i.e. the time between two crossings of the Sun’s path with the local meridian, is not constant either.
This variation, which we can call the *ellipse effect*, produces a sine wave variation of time (in relation to standard time) with an amplitude of 7.66 minutes and a period of one year.

![Figure 24](image)

**Figure 24:** Annually periodic difference between solar time and standard time due to ellipse effect

b) There is a second effect due to the tilt of the Earth axis of about 23.45° in relation to the normal of the orbit plane or the tilt of the ecliptic plane with respect to the equatorial plane. The following heuristic construct may be helpful to understand this effect: Imagine a planet P whose axis is parallel to the normal of the orbit plane. Imagine additionally that exactly once a year P rotates around its axis in the same sense as it rotates around its sun. In this case, the Sun will always be at the meridian at a certain longitude $L$. Now, incline the axis of P relatively to the normal of the orbit plane (for instance to an angle of 23.45° as happens with the Earth) and look what happens to the position of the Sun in relation to the longitude $L$. Except for four points at the orbit, the Sun will not be any more at the meridian. That means that in the segments between the four points (at spring and autumn equinox and at summer and winter solstice) there will be a difference between solar time and regular standard time. This effect, which is caused by the inclination of the Earth axis, is sometimes called the *projection effect.*

---

32 We consider here a planet with a circular orbit.
The segments between winter solstice and spring equinox and between summer solstice and autumn equinox are geometrically equal as well as the segments between spring equinox and summer solstice and between autumn equinox and winter solstice. Accordingly, the tilt of the Earth axis produces a regular variation with a period of a half year. The amplitude of the sine wave variation amounts to 9.87 minutes.

Figure 25: The projection effect

The segments between winter solstice and spring equinox and between summer solstice and autumn equinox are geometrically equal as well as the segments between spring equinox and summer solstice and between autumn equinox and winter solstice. Accordingly, the tilt of the Earth axis produces a regular variation with a period of a half year. The amplitude of the sine wave variation amounts to 9.87 minutes.

Figure 26: Half-year periodic difference between solar time and standard time due to projection effect

So, even disregarding the divergence of the solar time from the standard time because of the difference between local longitude and reference longitude, there is a difference between solar time and standard time, which can be expressed as the sum of the two explained geometric effects, the ellipse effect and the projection effect.
Figure 27 represents this sum graphically. As illustrated in the graphic, at its maximum the difference between solar time and local mean time amounts to more than 16 minutes. This means that an error of about 4° can result (according to the hour angle formula) if the two mentioned effects are not taken into consideration, which would be unacceptable in technical contexts such as the orientation of the mirrors in CSP plants.

Taking into consideration that there are differences between standard time and solar time at a given location due to, first, the existence of extended time zones and, second, the mentioned geometrical effects, we can distinguish between three types of times: local standard time, local mean time and solar time.

**Local standard time** is the official time in a given time zone.

**Local mean time** is a timescale that has a constant difference to the local standard time. This constant difference exists because of the difference between the reference longitude of the time zone and the longitude of the respective location within the time zone. The local mean time for a given location is valid for all locations at the same longitude. A *longitude correction* expresses the difference between local standard time and local mean time.

**Solar time** is a time scale according to which the Sun always crosses the meridian (in relation to a given location) exactly at noon. The so-called *equation of time* expresses the difference between local mean time and solar time. According to what was explained above, this difference is not constant but varies with an annual period.

<table>
<thead>
<tr>
<th>longitude correction</th>
<th>equation of time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Standard Time</td>
<td>Local Mean Time</td>
</tr>
</tbody>
</table>

The following table shows some important characteristics of the three time systems.
At first we will concretize the longitude correction, which accounts for the mentioned difference between the location longitude and the reference longitude of the corresponding time zone and permits to convert local standard time in local mean time and vice versa. As we know that the Sun needs 4 minutes to traverse an angle of 1°, this correction will be

\[ \Delta t = (L_r - L_{loc}) \left( 4 \text{ min/1°} \right) \]  

(25)

where \( L_r \) is the time zone reference longitude and \( L_{loc} \) the local longitude. Additional considerations may be necessary if the respective standard time contains changes between summer time and normal time. In this case, an additional difference of one hour has to be taken into account.

Second, the equation of time, which expresses the difference between local mean time and solar time and whose graphical representation is to be seen in figure 27, is the following:

\[ E = 180 \cdot \frac{4}{\pi} \left( 0.000075 + 0.001868 \cos d - 0.032077 \sin d - 0.014615 \cos 2d - 0.0409 \sin 2d \right) \text{ min} \]  

(26)

where \( d = 360° \cdot (\text{DoY} - 1)/365 \) and DoY = day of year.\(^{33}\)

Combining longitude correction and equation of time, solar time \( t_{so} \) can be derived from standard time \( t_{std} \) and vice versa:

\[ t_{so} = (L_r - L_{loc}) \left( 4 \text{ min/1°} \right) + E + t_{std} \]  

(27/1)

\[ t_{std} = -(L_r - L_{loc}) \left( 4 \text{ min/1°} \right) - E + t_{so} \]  

(27/2)

These equations are valid under the following longitude definition: Longitudes west of Greenwich are positive and longitudes east of Greenwich are negative, i.e. the Moroccan capital Rabat is situated at the longitude \( 6° \) and Cairo is situated at the longitude \(-31°\).

Now we have the instruments to calculate quite exactly the angular position of the Sun at a certain time in relation to any geographical position and to determine the incident angle of solar beam radiation at any plane at any geographical point.

3.4 Direction of direct solar radiation

For the calculation of the direction of solar beam radiation we need to determine the Sun position in relation to the observers’ horizontal system for the respective site and for a given time. In order to achieve this, we determine the equatorial coordinates $\delta$ and $\omega$ of the Sun’s position. For the calculation of the declination we need only the DoY, for the calculation of the hour angle we need the solar time. For the calculation of the solar time we need to know the longitude $L_{loc}$ of the location, the day of the year and the standard time $t_{std}$. Subsequently, we determine the Sun’s position in horizontal coordinates in relation to the observer’s location, taking into account its latitude $\Phi$.\(^{34}\) The solar altitude angle $\alpha_s$ is, according to (24/1):

$$\sin \alpha_s = \cos \Phi \cos \delta \cos \omega + \sin \Phi \sin \delta$$

In the same way we can indicate the zenith angle $\theta_z = 90 - \alpha_s$, which is identical to the angle of incidence $\theta$ of the solar beam radiation on a horizontal surface:

$$\cos \theta = \cos \theta_z = \sin \alpha_s = \cos \Phi \cos \delta \cos \omega + \sin \Phi \sin \delta \quad (28)$$

In order to calculate the azimuth angle of the Sun, we refer back to (24/2):

$$\cos \gamma_s \cos \alpha_s = \sin \Phi \cos \delta \cos \omega - \cos \Phi \sin \delta$$

$$\begin{align*}
\cos \gamma_s \cos \alpha_s &= \frac{\cos \phi \sin \phi \cos \delta \cos \omega - \cos \phi \cos \delta \cos \omega - \cos \phi \sin \delta \sin \omega}{\cos \phi} \\
&= \frac{\cos \phi \sin \phi \cos \delta \cos \omega - (1 - \sin^2 \phi) \sin \delta}{\cos \phi} \\
&= \frac{\sin \phi \cos \delta \cos \omega + \sin \phi \sin \delta \sin \omega}{\cos \phi}
\end{align*}$$

This expression can be simplified making use of (24/1):

$$\begin{align*}
\cos \gamma_s \cos \alpha_s &= \frac{\sin \phi \sin \alpha_s \sin \delta}{\cos \phi} \\
\cos \gamma_s &= \frac{\sin \phi \sin \alpha_s \sin \delta}{\cos \phi \cos \alpha_s}
\end{align*} \quad (29)$$

Because of the symmetry of $\cos x$ with respect to $x = 0$, this equation does not contain any information about the algebraic sign of $\gamma_s$. But, it is important to know it, because it indicates whether the Sun is east of south by $|\gamma_s|$, being $\gamma_s$ negative (in accordance with the agreement to determine angles east of south as negative and west of south as positive), or west of south by $|\gamma_s|$, being $\gamma_s$ positive. However, the algebraic sign can be fixed quite easily: Before solar noon $\gamma_s$ should be negative, because the Sun is east of south, and after solar noon $\gamma_s$ should be positive, because the Sun is west of south. The same is valid for the hour angle $\omega$, so that we can derive the arithmetic sign of $\gamma_s$ from the arithmetic sign of $\omega$:

\(^{34}\) In accordance with the definition of the declination $\delta$, which was defined as positive during northern spring and summer and negative during southern spring and summer, $\Phi$ is positive in the northern hemisphere and negative in the southern hemisphere.
where the sign function \( \text{sign}(\omega) \) is equal to +1 if \( \omega \) is positive and it is −1 if \( \omega \) is negative.

If we take the incidence angle \( \theta_z \) instead of the solar altitude angle \( \alpha_z \), then (30/1) changes to:

\[
\gamma_z = \text{sign}(\omega) \left| \cos^{-1} \left( \frac{\sin \phi \sin \theta_z - \sin \delta}{\cos \phi \cos \alpha_z} \right) \right| 
\]  

However, as mentioned above, one has to bear in mind that all of these formulae are based on the position of the Sun in relation to the Earth only. They do not take into account any deviation of light by the curved and layered atmosphere, which changes the apparent Sun position slightly. This effect can be important for CSP systems as even small errors in tracking angles lead to a loss in radiation collection efficiency because parts of the sunlight will not hit the absorbers any more. The corresponding numerical formulae that account for these effects will be unequally more complex and would go beyond the scope of this lecture.

### 3.5 Incidence angles on tilted surfaces

We consider now a tilted plane at a given geographical location and want to determine the incidence angle of the beam radiation on it. The incidence angle depends on the following parameters: geographical position (latitude and longitude), time (day of year as well as time of day) and plane orientation (tilt angle and azimuth angle).

Tilt angle \( \beta \) and an azimuth angle \( \gamma \) of the plane will be defined the following way: Take a horizontal surface and incline it to the south (in positive direction) or to the north (in negative direction) by \( \beta \) and rotate it then to the west (in positive direction) or to the east (in negative direction) by \( \gamma \).
Taking into consideration that the time will be given in standard time, we will have to determine first the corresponding solar time (from day of year, time of day, longitude). In a second step we can calculate the hour angle $\omega$ (from solar time) and declination $\delta$ (from day of year). With these angles and with the plane tilt angle $\beta$ and the plane azimuth angle $\gamma$ we can determine the incidence angle $\theta$ according to the following formula\textsuperscript{35}:

$$\cos \theta = \frac{\sin \delta \sin \Phi \cos \beta}{-\sin \delta \cos \Phi \sin \beta \cos \gamma + \cos \delta \cos \Phi \cos \beta \cos \omega + \cos \delta \sin \Phi \cos \beta \cos \gamma \cos \omega + \cos \delta \sin \beta \sin \gamma \sin \omega}$$ \hspace{1cm} (31)

In some special cases this formula adopts a simpler form. First, if we consider horizontal surfaces, then the incidence angle will be independent of the plane azimuth angle $\gamma$. Thus, the terms that contain the azimuth angle will disappear and the equation takes the form of (28) (with the incidence angle being identical to the zenith angle):

$$\cos \theta = \cos \theta_z = \cos \Phi \cos \delta \cos \omega + \sin \Phi \sin \delta$$

Second, for vertical surfaces ($\beta = 90^\circ$) facing towards south in the northern hemisphere ($\gamma = 0^\circ$) the equation transforms into:

$$\cos \theta = -\sin \delta \cos \Phi + \cos \delta \sin \Phi \cos \omega$$ \hspace{1cm} (32/1)

\textsuperscript{35} Duffie/Beckman 2006, 14
while it takes the form \( \cos \theta = \sin \delta \cos \Phi - \cos \delta \sin \Phi \cos \omega \) for vertical surfaces facing towards north in the southern hemisphere.

Third, for inclined surfaces in the northern hemisphere facing towards the equator with a tilt angle equal to the latitude angle \( \beta = \Phi, \gamma = 0^\circ \) the equation will be simplified to:

\[
\cos \theta = \cos \delta \cos \omega \tag{32/2}
\]

(for solar noon, where \( \omega = 0^\circ \), it takes the very simple form \( \theta = |\delta| \).)

The indicated formula permits also to calculate the hour angles of sunrise and sunset \( \omega_{ss} \) \( (\theta = 90^\circ) \). As the plane tilt is of no interest in this case, we take a horizontal surface \( (\beta = 0^\circ) \) and we get the relatively simple relation

\[
0 = \cos \Phi \cos \delta \cos \omega + \sin \Phi \sin \delta
\]

and consequently

\[
\cos \omega_{ss} = -\tan \delta \tan \Phi. \tag{33}
\]

However, bear in mind that (33) is based on the moment when the centre of the Sun is at the horizon. In practice, sunrise and sunset are defined as the times when the upper limb of the Sun is on the horizon, i.e. \( \theta \) is not \( 90^\circ \) but slightly more, such that in reality \( |\omega_{ss}| \) will be a bit higher than indicated by (33). Disregarding this detail, we can use (32) to calculate the day length for any day at any latitude. In order to do that, we calculate the declination according to (22), put it in (33) and determine the two values for \( \omega_{ss} \). The difference of these two values allows calculating the day length \( t_s \) according to (23):

\[
t_{\text{day}} = (\omega_{ss,2} - \omega_{ss,1}) \frac{1}{15} \text{ h} \tag{34}
\]

With (22) we can transform (32) into an expression as a function of the latitude and the day of the year:

\[
\cos \omega_{ss} = -\tan \left[ 23,45^\circ \sin \left( 360^\circ \frac{284 + \text{DoY}}{365} \right) \right] \tan \Phi. \tag{35}
\]

The incidence angle in (30) and in its subsequent simplified forms is indicated in the cosine form. This is not just for aesthetic reasons, avoiding ugly arccos or \( \cos^{-1} \) signs, but it has also the advantage to relate the incidence angle directly to the irradiance on a plane, yet the beam radiation arriving with an incidence angle of \( \theta \) at a plane provokes an irradiance of \( G_{bt} = \cos \theta \cdot G_{bn} \) where \( G_{bn} \) is the irradiance on a plane in normal orientation to the radiation (cosine effect).
3.6 Tracking angles of mirror systems

All radiation concentrating systems have in common that they need some tracking system if they shall be used continuously. Direct radiation changes continuously its direction in relation to the horizontal coordinate system of a given location and, consequently, in relation to an optical system fixed on the terrestrial surface. As the radiation has to enter the collector system in a determinate direction, a tracking system is required in order to maintain the mirrors in line with the incident direct radiation.

There are differences between point-focusing and line-focusing systems. Point-focusing systems (solar dish and solar tower systems) need a two-axis tracking system, while line-focusing systems (parabolic trough and linear Fresnel systems) only require one-axis tracking.

Point-focusing systems: A determination of the tracking angles of dish-Stirling systems can be very short, yet, they are identical to the angles of the beam radiation as they were determined in the last section. (Always bear in mind though, that the actual formulae used in practice, which account also for atmospheric effects, are much more complicated.)

![Diagram showing tracking angles at point-focusing systems](image)

Figure 29: Tracking angles at point-focusing systems

The following angles can be used:

The collector zenith angle $\theta_z$, which indicates the inclination of the concentrator in relation to the vertical is identical to the solar zenith angle $\theta_z$ (and the incidence $\theta$ of the solar beam radiation on a horizontal surface). It is calculated, according to (28), as follows:

$$
\cos s = \cos \theta_z = \cos \Phi \cos \delta \cos \omega + \sin \Phi \sin \delta
$$

where $\Delta \cos (s)$ is equal to +1 if $\omega$ is positive, and −1 if $\omega$ is negative.

The azimuth angle $\gamma$ of the concentrator is identical to the solar azimuth angle $\gamma_s$. It is calculated, according to (30/2), as follows:

$$
\gamma_s = \text{sign}(\omega) \left| \cos^{-1} \left( \frac{\sin \Phi \cos \theta_z - \sin \delta}{\cos \Phi \sin \theta_z} \right) \right|
$$

where the sign function $\text{sign}(\omega)$ is equal to +1 if $\omega$ is positive, and −1 if $\omega$ is negative.

The heliostats of solar tower power plants also require two-axis tracking. However, the situation is more complicated than for a dish/Stirling system because the tracking not only depends on the geographical position of the heliostat and the Sun’s position, but also on the position of the solar tower and its receiver in relation to the heliostat.
Taking into consideration that the angle of incidence on the mirrors equals the angle of reflection, for the azimuth angle and the altitude angle of the normal of the heliostat the following holds:

The altitude angle of the normal of a heliostat is the arithmetic middle of the altitude angle of the line from the heliostat to the receiver and the solar altitude angle:

$$\alpha_{hs} = \frac{\alpha_{st} + \alpha_s}{2}$$

The azimuth angle of the normal of the heliostat is the arithmetic middle of the azimuth angle of the line from the heliostat to the receiver and the solar azimuth angle:

$$\gamma_{hs} = \frac{\gamma_{st} + \gamma_s}{2}$$

$\alpha_{hs}$ and $\gamma_{hs}$ are determined at given solar field and solar tower geometries. $\alpha_s = 90^\circ - \theta_2$ is determined by means of equation (36) and $\gamma_s$ is determined according to equation (37).

Line-focusing systems: The tracking of line-focusing systems requires only one tracking axis. Theoretically, the tracking axis can be oriented in any direction. However, usually east-west alignment, which tracks the Sun from north to south, and north-south alignment, which tracks the Sun from west to east, are considered. East-west alignment has the advantage that comparably little collector adjustment is required during the day. At noon time the full aperture always faces the Sun. However, a disadvantage is that the collector performance is reduced greatly during the early and late hours of the day due to large incidence angles. North-south alignment has the advantage to have a much more equilibrate daily collector performance. Generally, the cosine loss is higher at noon than in
the morning or evening hours. The differences between summer and winter concerning the collector performance are larger in north-south alignment than in east-west alignment.

In commercial applications, a horizontal north-south alignment is used. Higher annual energy yields and the mentioned lower daily power variations are the reasons for this decision. The orientation can be described by the collector zenith angle $s$, which is the angle between the optical plane and the line to the zenith, and the azimuth angle $\gamma$, which indicates the orientation of the mirror aperture in relation to the horizon, where south=0° and west=90°. The tracking angle is then the angle $s$, which is calculated as follows:

$$\tan s = \tan \theta_z |\cos (\gamma - \gamma_s)|$$

(38)

where $\theta_z$ is the solar zenith angle, $\gamma_s$ the solar azimuth angle. For $\gamma$, the following determination holds: $\gamma = -90^\circ$ if $\gamma_s < 0^\circ$ and $\gamma = 90^\circ$ if $\gamma_s > 0^\circ$, which means that the mirror aperture is oriented to the east in the morning and to the west in the afternoon.

**Figure 31:** Tracking angles at line-focusing systems with north-south alignment

In the case of east-west alignment, the tracking angle $s$ is calculated as follows:

$$\tan s = \tan \theta_z |\cos \gamma_s|$$

(39)

For $\gamma$, the following determination holds: $\gamma = 0^\circ$ if $|\gamma_s| < 90^\circ$ and $\gamma = 180^\circ$ if $|\gamma_s| > 90^\circ$, which means that the mirror aperture is oriented to the south if the Sun is south of the east-west line and to the north if the Sun is north of the east-west line (which happens between spring equinox and autumn equinox in the early morning and in the late evening).

**Figure 32:** Tracking angles at line-focusing systems with north-south alignment

---

36 This is the case for locations not too close to the equator and rather in winter. Anyway, concentrating solar power plants normally cannot be located close to the equator because the direct solar radiation is less because of high cloud indexes.
3.7 Incidence angles on mirror systems

The incidence angle on solar dishes is always 0° because of the two-axis tracking. The incident angle on line-focusing parabolic trough systems with one-axis tracking, on the contrary, depends on the collector alignment and on the Sun position. For the common north-south alignment, the incident angle is calculated as follows:

$$\cos \theta = \sqrt{\cos^2 \theta_z + \cos^2 \delta \cdot \sin^2 \omega}$$  \hspace{1cm} (40)

where $\theta_z$ is the solar zenith angle, $\delta$ the declination and $\omega$ the hour angle. For the east-west alignment, the incidence angle is calculated as follows:

$$\cos \theta = \sqrt{1 - \cos^2 \delta \cdot \sin^2 \omega}$$  \hspace{1cm} (41)
4 Radiation concentration

4.1 Radiation concentration on parabolic mirrors

Parabolic mirrors have a focal point or a focal line. Paraboloids (solar dishes) have a focal point and parabolic troughs have a focal line.

Radiation which enters parallel to the optical axis of a paraboloid mirror or within a plane parallel to the optical axis plane\(^{37}\) of a parabolic trough is reflected a way that it passes through the focal line. Reduced to a two-dimensional problem: A parabola has a focal point, which means that radiation that arrives parallel to the optical axis is reflected a way that it passes though the focal point. Figure 33 illustrates the geometric relations. It demonstras the cross section of a parabolic mirror in a coordinate system. The parabolic mirror follows the graph of the function \( y = px^2 \). The red lines represent the light entering the mirror in rays parallel to the axis of the parabola and crossing the axis, after reflection, at the focal point F.

In the following we want to show that such a parabola really has a focal point. Therefore, we take one arbitrary ray of light that is parallel to the axis of the parabolic mirror and we determine where the reflected ray crosses the axis. We show that this point does not depend on the point were the ray hits the mirror and we thereby demonstrate that all rays parallel to the parabola’s axis pass through this point after reflection.

We take the ray that hits the mirror, or the tangent \( \overline{EB} \), at the point E under the incident angle \( \alpha \) leaving E under the same angle. \( \angle BED \) is the vertical angle of the incident angle and therefore it also equals to \( \alpha \).

Now we determine the point B on the x-axis, i.e. the point of intersection of \( \overline{EB} \) with the x-axis.

\( \angle \) The expression “optical plane” refers to the plane that contains the optical axes of all (parabolic) cross sections of the trough.

Figure 33: Path of rays parallel to the optical axis of a parabolic mirror

\( \angle \)
E is on the graph of the function \( y = px^2 \). Consequently, the coordinates of E, \((x_E, y_E)\), can be determined as \((x_E; px_E^2)\). The gradient of \( EE \) is the gradient of the function \( y = px^2 \) at the point E, i.e. \( \frac{dy}{dx}(x_E) = 2px_E \). According to this, \( EE \) has the analytical form \( y = 2px_Ex + b \). Filling \((x_E; px_E^2)\) in this equation, we get \( b = -px_E^2 \) and, consequently, \( y = 2px_Ex - px_E^2 \). For B, which is the point of intersection of this line with the x-axis, i.e. for \((x_B; 0)\) we get \( x_B = x_E/2 \). So, B divides \( AD \) in two equal parts. We can draw the line through F and B and get the point C such that \( \triangle ABF \) and \( \triangle BCD \) are congruent. Furthermore, these triangles are similar to \( \triangle BDE \). Now, because of this similarity it is \( \frac{AF}{AB} = \frac{BD}{DE} \) and with \( AB = BD = x_E/2 \) and \( DE = px_E^2 \) we get

\[
\frac{AF}{AB} = f = \frac{1}{(4p)}.
\]

We see that the point of intersection of the reflected ray with the x-axis is independent from the incident point E. That means there is a focal point where all incident rays meet, that enter the mirror parallel to its axis, and the focal length of a parabola (from the vertex) is \( f = 1/(4p) \). Consequently, the parabola’s analytic representation can be expressed as

\[
y = \frac{1}{4f}x^2.
\]

So, parabolic mirrors concentrate radiation. Adding a third dimension \( z \) to the two-dimensional x-y-plane in figure 33 there are two ways to design a three-dimensional parabolic mirror. First, the parabola can be elongated in the \( z \)-direction forming a parabolic trough. In this case the focal point is also elongated into a focal line. Second, the parabola can be rotated around its axis thus generating a paraboloid, which concentrates the radiation in a point.

![Figure 1.34: Concentrating parabolic mirrors, point-concentrating (left) and line-concentrating (right)](image)

### 4.2 Alternative radiation concentration geometries for CSP systems

There are other important concentration geometries that are not derived directly from the described focusing properties of the parabola. We can distinguish, once more, between point-focusing and line-focusing systems.

An important point-focusing system are the so-called heliostat fields of solar tower plants with a number of mirrors that track the Sun such that the rays always hit a receiver area on a tower.
An important line-focusing system is the so-called Fresnel collector\(^{38}\) that consists of a number of elongated mirror segments that track the Sun such that the rays always hit a receiver in the focal line.

### 4.3 Theoretical maximum concentration of solar radiation

As shown in the preceding section, parabolic mirrors concentrate axis-parallel radiation in a focal point or in a focal line. Now, the following question arises: Is there a limitation of the possible concentration ratio at parabolic mirrors or is it possible to reach any concentration ratio? The concentration ratio \( C \) is defined as the ratio between the radiant flux after the concentration to the radiant flux before the concentration. In many cases, this ratio can be approximated by the ratio of the aperture area of the optical system, i.e. the area through which the radiation enters, to the minimum area through which the reflected radiation passes, i.e. to the area of the image of the radiation source at the point where the area of this image is minimal:

\[
C = \frac{\text{aperture area}}{\text{area of radiation source image}}
\]  

\(^{38}\) The name is derived from the Fresnel lenses, developed by the French physicist Augustin-Jean Fresnel, that have the characteristics to break a big lens into a set of concentric annular sections, which reduces the amount of material required compared to a conventional spherical lens with similar optical properties.
Taking into account that we deal with systems that concentrate the solar radiation incident on the aperture area onto an absorber, and supposing that the absorber surface covers just the Sun image, we can convert (43) also into the following form:

\[ C = \frac{\text{aperture area}}{\text{absorber area}} \]  

(44)

In our case, we have to consider the image of the Sun in the focal point – more exactly, as we will see in the following, in the focal plane – of the optical system. Now, taking into account that the preceding section demonstrated that there is (at least at a perfect parabolic mirror) a dimensionless focal point (and not an extended focal area) somebody could assume that it is possible to reach any concentration ratio, yet, the Sun’s image could be a dimensionless point. However, even with a perfect mirror, this is impossible, because the solar radiation does not arrive in exactly parallel rays and, consequently, the Sun’s image is not concentrated in the focal point that was calculated above but occupies a certain area around the focal point in the focal plane.\(^{39}\) Remember that the Sun-Earth distance is finite so that there is a angular spread of the direct solar radiation. The spread angle is called the solar beam angle and it amounts to \(\alpha_D = 32^\circ = 0.53^\circ\) (figure 37).\(^{40}\) The existence of a certain beam spread makes it impossible to concentrate the direct solar radiation that enters an optical system in one point. There are finite maximum concentration ratios.

![Figure 37: Solar beam angle](image)

\(\alpha_D = 32^\circ\)

There is a general law which permits the determination of this maximum concentration on the basis of the beam angle of the incident radiation: the law of the conservation of the \(\text{étendue}\) in an ideal optical system. An ideal optical system is a system in which there are no energy losses by extinction processes (absorption); the reflection coefficients of included mirrors are 1 and the transmission coefficient of the included optical medium is also 1. In such a system, as indicated in figure 38, and under the additional condition that in the optical path there is no change among different optical media with different refraction indexes, the product of the area of the aperture A and of the receiver \(A'\) and the solid angle in which the radiation propagates is constant. Or, what amounts to the same, the products of the areas A and \(A'\) and respective squares of the sine of the half beam angle \(\frac{\alpha}{2}\) and \(\frac{\alpha'}{2}\) are equal:

\[ A\sin^2\frac{\alpha}{2} = A'\sin^2\frac{\alpha'}{2} \]  

\(^{39}\) The focal plane is defined as the plane that includes the focal point above determined and is perpendicular to the parabola’s axis.

\(^{40}\) The following derivation of the maximal concentration ratio is taken from Kleemann/Meliß 1993, 109-111.
Taking into consideration the definition of the concentration ratio (44) and taking into account that 
\( \frac{\alpha_0}{2} = 16' = 0.267^\circ \) (half of the solar beam angle), the maximum concentration ratio, which is reached 
for \( \frac{\alpha''}{2} = 90^\circ \), amounts to:

\[
C_{\text{max}} = \left( \frac{A}{A'} \right)_{\text{max}} = \left( \frac{\sin^2 \frac{\alpha''}{2}}{\sin^2 \frac{\alpha_0}{2}} \right)_{\text{max}} = \frac{1}{\sin^2 0.267^\circ} = 46200. \tag{45}
\]

This theoretical concentration maximum is valid for an ideal three-dimensional concentrating system 
that concentrates the incident radiation in one spot. In a two-dimensional concentration system, which 
concentrates the incident radiation in a line, the product of area and the sine of half of the beam angle 
(instead of the square of the sine of half of the beam angle) is constant. Therefore, the maximum 
concentration ratio amounts to:

\[
C_{\text{max}} = \left( \frac{A}{A'} \right)_{\text{max}} = \left( \frac{\sin^{\frac{\alpha''}{2}}}{\sin^{\frac{\alpha_0}{2}}} \right)_{\text{max}} = \frac{1}{\sin 0.267^\circ} = 215. \tag{46}
\]

### 4.4 Theoretical maximum concentration ratio on parabolic mirrors

Now we will determine the theoretical maximum concentration ratio of the focal spot or the focal line 
in real parabolic systems. We will consider flat receivers in the focal plane (which is realistic for dish 
systems, but not for parabolic troughs).

At first we consider paraboloid mirrors. The Sun image in the focal plane is a fuzzy spot whose total 
size and form depend on the aperture of the mirror and on the range of the rim angle \( \psi \) (figure 39). 
The total image is composed of the individual images coming from each point \( P \) on the mirror’s 
surface. These individual images are ellipses whose form and size depend in the following way on the 
angle \( \psi \):
Figure 39: Focal spots at paraboloid mirror

\[ a = r_r \alpha_D \]  \hspace{1cm} (47)

\[ b = \frac{r_r \alpha_D}{\cos \psi} \]  \hspace{1cm} (48)

Considering all points at the distance \( r_r \) from the focal point F, the Sun image in the focal plane has a circular form with the diameter \( d_{im} = \frac{r_r \alpha_D}{\cos \psi} \).

Taking into account that the Sun images from all other points with a distance from F that is smaller than \( r_r \) are situated within this circular spot, we can assert that this circle indicates the total size of the Sun image of the whole paraboloid mirror. The Sun image covers the following area:

\[ A_{im} = \frac{\pi r_r^2 a_D^2}{4 \cos^2 \psi} \]  \hspace{1cm} (49)

The diameter \( d \) of the paraboloid mirror is related to the maximal value of \( \psi \) and to \( r_r \) as follows: \( d = 2 \rho_s \sin \psi \), and the aperture area amounts to:

\[ A_{ap} = \pi r_r^2 \sin^2 \psi \]  \hspace{1cm} (50)

The concentration ratio \( C \) is:

\[ C = \frac{A_{ap}}{A_{im}} = \frac{4}{a_D^2} \sin^2 \psi \cos^2 \psi \]  \hspace{1cm} (51)

and with \( \alpha_D = 32' = 0.5333 \degree = 0.009308 \text{rad} \):

\[ C = 46200 \sin^2 \psi \cos^2 \psi \]  \hspace{1cm} (52)

The maximum value is reached for \( \psi = 45 \degree \).
This value is smaller than the theoretical concentration ratio of 46200 that was indicated in equation (45). The difference is explained by the fact that equation (45) refers to punctual maxima, while equation (53) refers to the radiation concentration averaged over the whole Sun image. The local concentration ratios within the focal spot vary so that the maximum punctual concentration ratio is higher than the average concentration ratio.

In the case of a parabolic trough of the length $l$, the aperture area is $A_{ap} = 2lr\sin \psi$ and the Sun image on a plane receiver has the area $A_{im} = \frac{r^2 \alpha \eta}{\cos \psi}$. The corresponding concentration ratio is:

$$C = \frac{A_{ap}}{A_{im}} = \frac{2\sin \psi \cos \psi}{\alpha \eta} = 215 \sin \psi \cos \psi$$  \hspace{1cm} (54)

The maximum value is reached again for $\psi = 45^\circ$:

$$C_{max} = 215 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 107.5$$  \hspace{1cm} (55)

Once more, this value is lower than the value in equation (46) because in that equation the punctual concentration maximum is indicated while equation (55) indicates the average concentration ratio within the Sun image.

The calculated values are valid only for ideal mirrors and the specified geometry. Real systems with paraboloid mirrors reach concentration rates of about 2000-6000.\(^{41}\) Real parabolic troughs reach an average concentration ratio of 82.\(^{42}\)

As mentioned above, the irradiance on the focal spot is not homogenous. In case of a paraboloid concentration geometry, the centre of the focal spot has a higher irradiance than the outer parts. More exactly, the irradiance shows a Gaussian distribution as illustrated by the graph in figure 40, which demostrates the irradiance distribution in the focal spot of an experimental device at DLR.

\(^{41}\) See also Kleemann/Meliß 1993.

\(^{42}\) The values for the parabolic troughs result if the ratio of the aperture width of the collector to the receiver diameter is used to determine the concentration ratio. Many producers of parabolic troughs indicate this value. However, it has to be taken into account that it would be more exact to calculate the concentration ratio not in relation to the absorber diameter but in relation to the absorber circumference (which would render lower concentration ratios).
For high temperature applications it could be decided to use only the inner part of the focal spot, which has higher concentration ratios.

The fact that real concentration ratios are below the calculated theoretical maximum concentration ratios is implied by the following aspects:

- geometrical imperfections:
  - surface imperfections of the mirror (microscopic and macroscopic)
  - orientation error of the reflector

- material limitations:
  - mirror reflection coefficients below 1
Figures 41 and 42 illustrate the geometrical imperfections. Macroscopical surface aberrations of the mirror (slope errors) by the angle $\psi$ result in a ray aberration of $2\psi$ (figure 41).

Microscopical surface errors may result in a widening of the beam spread of the reflected radiation. Additionally, there are material limitations, which provoke in this case that the reflection coefficient is below 1. That means that a certain part of the incoming radiation will not be reflected but absorbed by the mirror material or transmitted.

4.5 Maximum theoretical absorber temperature

The maximum temperature that can be reached on the absorber depends on the concentration ratio.\textsuperscript{43} According to the Stefan-Boltzmann law, the radiant emittance of a black body surface with the area $A$ and the temperature $T$ is calculated as follows

$$P = A \sigma T^4.$$\textsuperscript{56}

Thus, the radiant emittance of the Sun ($A = 4\pi r_S^2$, $T = T_S$) approximately (in due consideration of the comments in 2.1) amounts to

$$P_S = 4\pi r_S^2 \sigma T_S^4.$$\textsuperscript{57}

According to the determination of the solar constant

$$G_{SC} = \frac{P_S}{4\pi r_{SE}^2}.$$\textsuperscript{58}

\textsuperscript{43} The following derivation of the maximal concentration ratio is taken from Kleemann/Melß 1993, 111-114.
where $r_{SE}$ is the Sun-Earth-distance, and without taking into consideration atmospheric influences (!), the received power on the collector aperture ($A = A_{ap}$) is

$$
\dot{Q}_{ap} = A_{ap} \frac{P_S}{4\pi r_{SE}^2} = A_{ap} A \pi r_S^2 \sigma T_S^4 / 4\pi r_{SE}^2
$$

$$
= A_{ap} \sigma T_S^4 \frac{r_S^2}{r_{SE}^2}.
$$

(59)

The radiant power of the absorber ($A = A_{ab}$, $T = T_{ab}$), considering the absorber as a black body, amounts to

$$
P_{ab} = A_{ab} \sigma T_{ab}^4.
$$

(60)

The maximum absorber temperature is reached when the radiant power of the absorber equals the received power on the collector aperture, i.e. when the radiation balance of the absorber is zero:

$$
P_{ab} = A_{AB} \sigma T_{ab}^4 = \dot{Q}_{ap} = A_{AP} \sigma T_S^4 \frac{r_S^2}{r_{SE}^2}.
$$

(61)

As $\alpha_D$ is quite small, we can approximate $\alpha_D \approx \tan \alpha_D = 2r_z/r_{SE}$ and it follows $\alpha_D^2/4 = r_z^2/r_{SE}^2$. Taking into account, additionally, equation (44) we get

$$
r_S^2/r_{SE}^2 = 1/C_{\text{max}}.
$$

(62)

With (62), (61) can be transformed (including further simplification) into

$$
A_{AB} T_{ab}^4 = A_{AP} T_S^4 \frac{1}{C_{\text{max}}}
$$

(63)

With $A_{AP}/A_{AB} = C$ (equation 44) equation (63) can be transformed into:

$$
T_{ab} = T_S \left( \frac{C}{C_{\text{max}}} \right)^{\frac{1}{4}} = 5780K \left( \frac{C}{46200} \right)^{\frac{1}{4}}
$$

(64)
Equation (64) shows the dependence of the absorber temperature on the concentration ratio. The higher the concentration ratio is, the higher is the temperature of the absorber. However, this equation is quite theoretical. First, it considers black bodies, although we know that no real object is a black body, nor the Sun neither the absorber. Selective coatings on absorbers in solar thermal systems, in particular, may reduce the radiant emittance and provoke, thus, higher absorber temperatures. Second, absorbers may be protected by claddings, for instance glass covers or tubes, that reflect or absorb and reemit a part of the radiation emitted by the absorber and that permit, thus, higher absorber temperatures. Third, only radiation is considered. Other heat exchange processes are not taken into account. However, in the case of a colder environment, heat conduction and convection may have the effect that the maximum temperature is never reached. Fourth, atmospheric influences are not considered. As we have seen, atmospheric influences reduce the solar irradiance, such that they tend to reduce the absorber temperature. Nevertheless, one information included in equation (64) is valid independently from all further considerations: The theoretical maximum temperature at $C = C_{\text{max}}$ would be $T_{AB} = T_S = 5780\, \text{K}$. A higher temperature would mean a free energy flow from a body with a lower to a body with a higher temperature, and this is impossible according to the second law of thermodynamics.

---

**Figure 43:** Theoretical maximum temperature at different concentration ratios (Kleemann/Meliß: 1993, 113)

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44 This is what happens to the Earth. Taking the Earth without any protecting atmosphere, its mean temperature at a radiation balance of zero would be much lower than it is actually: around -18°C. Thanks to atmospheric influences the real mean temperature is higher.
5 Radiation measurement

Solar power projects need reliable local radiation data for the project site. There are different possibilities to get such data. Radiation can be measured by ground measuring or by satellite measuring. Radiation data bases are available. In this section we will concentrate on radiation measurement. At the end, some available data bases are presented.

Ground and satellite measurements have different functional characteristics. Ground measurement can be done by a CSP project developer himself; satellite measurement requires larger measurement programmes. Ground measurements are local (with the possibility of spatial interpolation); satellite measurements cover large regions and allow the comparison of many possible sites. Ground measurements require large time spans in order to achieve reliable results; satellite measurement programmes often count with available long-term data bases. Ground measurements are very exact if executed in an appropriate way; satellite measurements cannot reach the exactness of ground measurements.

In many cases, an adequate way of solar resource assessment may be a predetermination of possible CSP plant sites on the basis of satellite data and the subsequent ground measurement on a predetermined site in order to acquire more exact data for the selected site.

**Figure 44:** Two possibilities of radiation determination: ground measurement and satellite measurement

5.1 Ground measurement

There are different radiation measurement instruments and techniques for ground measurement, the application of which depends on the measurement aim. There are different instruments for the measurement of global radiation, direct radiation and diffuse radiation, and there are different instruments for the measurement of the radiation in different wavelength ranges. A differentiation of measurement methods and instruments in relation to wavelength ranges is less important in the context
of CSP and can be neglected here. A differentiation of measurement procedures of direct, diffuse and global radiation, on the contrary, is very important.

Figure 45: Global radiation measurement with pyranometer (left) and infrared radiation measurement with a pyrgeometer (right) (source: Gengenbach Messtechnik)

Diffuse radiation measurement instruments cover the whole hemisphere above the measurement point (180 degrees viewing angle) and are shaded from the Sun. The detector is placed horizontally. Direct radiation measurement instruments are oriented directly to the Sun so that the detector surface is always perpendicular to the incoming direct radiation. Global radiation measurement instruments receive both the diffuse and direct component on a horizontally placed detector.

One standard instrument for measuring the global irradiance is a thermopile-type pyranometer, which measures the irradiance in the wavelength range of solar radiation (typical measurement range: from 300 to 3000 nm). It uses thermopile detectors and glass or quartz domes.\(^{45}\) The dome has a double function. First, the filter material reduces the wavelength range of the registered radiation, excluding the ambient infrared radiation.\(^{46}\) Second, it protects the device against different environmental influences, e.g. pollution. The form of a dome allows equal transmittance of the solar radiation from all angles. Radiation measurement instruments normally have integrated bubble levels and adjustable levelling feet for horizontal positioning. Additionally, they dispose of a drying cartridge filled with silica gel desiccant, which has the function to avoid condensation on the inner dome surface.

The mode of operation of a thermopile-type pyranometer is based on the thermoelectric effect. The thermopile has a black surface coating, which is a non-spectrally selective paint with a very high absorption coefficient (about 0.97). It consists of a large number of thermocouple junction pairs, which are connected electrically in series. One junction responds to the incident radiant flux warming up according to the incident radiation. The other junction is connected to a heat-sink, which is the pyranometer body itself. Between them there is a thermal resistance. Thermocouples convert the temperature difference into a voltage. The ratio of the generated voltage \(U\) and irradiance \(G\) is

\[^{45}\text{See www.kippzonen.com, www.rg-messtechnik.de.}\]

\[^{46}\text{In the case of pyrgeometers, i.e. instruments for the measurement of infrared radiation, coated silicon lets pass only radiation with a wavelength between 3μm and 50μm.}\]
constant, with the sensitivity of the radiometer $S$ as the proportionality factor, such that the measured voltage is easily translated into irradiance:

$$S = \frac{V}{A}$$

measured radiation range: $0.3 \, \mu m \leq \lambda \leq 3 \, \mu m$

functional principle

- black absorber surface under the glass dome is warmed up by incoming radiation
- thermal connection between the edge of the absorber and the outer pyranometer body
- temperature difference between the centre and the edge of the absorber is measured with a thermopile

Pyranometers can also be used to measure diffuse radiation instead of global radiation. Therefore, shadow rings or shading balls are applied, which shade continually the sensitive surface from direct solar radiation, permitting only diffuse radiation to pass. Such measurements with shading elements permit additionally to determine direct radiation considering that direct radiation is the difference between global and diffuse radiation. Shadow rings are the most economic solution for diffuse radiation measurement. They are inclined to the horizontal plane by $90^\circ - \Phi$, where $\Phi$ is the geographical latitude, adopting thus a parallel orientation to the equatorial plane. Adjustment is required every several days by parallel displacement of the shadow ring along the two parallel fixing rods. Shadow rings also block a small part of the diffuse radiation, which has to be taken into account in a respective correction.
Contrary to shadow rings, shading balls shield only the Sun and its aureole. However, they need a complete Sun tracking system to follow the Sun.

Another instrument designed especially to measure direct normal beam radiation is a **pyrheliometer**. Its field of view is limited to 5°. That means that it “sees” the Sun and its aureole and not the rest of the sky. This is achieved by the shape of the collimation tube, the shape of the aperture and the detector design. The front aperture is fitted with a quartz window as a filter that lets pass solar radiation with a wavelength between 200nm and 4000nm. The mode of operation is analogue to the one of pyranometers. Pyrheliometers must be oriented accurately and permanently to the Sun. That’s why it is necessary to use automatic Sun trackers.
The mentioned instruments permit to measure the radiation with a high accuracy. The error is less than 2%. However, in order to achieve this accuracy, quite an intensive maintenance is required. The instruments are very sensitive to soiling. Daily cleaning and a permanent local professional surveillance are necessary in order to obtain reliable results.

An alternative instrument, which allows the simultaneous measurement of global, direct and diffuse radiation and which needs less maintenance work, is the **rotating shadow band pyranometer**. One monthly revision and cleaning is sufficient. Its accuracy is slightly lower than that of a well maintained and cleaned thermopile-based measurement system (error of about 3%). Contrary to the latter, a rotating shadow band pyranometer works with a photocell that converts the radiation into electric signals. It is equipped with a shadow band that rotates in regular time intervals around the photocell and casts thereby momentarily a shadow over it. This permits to determine global radiation as well as diffuse radiation. Global radiation is measured when the shadow band does not block the direct radiation, and diffuse radiation is measured at the moment when it prevents that the direct radiation reaches the photocell.
Direct radiation is determined as the difference of global and diffuse radiation. In commercially available systems, the shadow band moves once per minute over the sensor, taking about one second for this motion. During this period the sensor signal is sampled about 1000 times. The lowest readings occur when the sensor is completely shaded from the Sun and when the instrument reads only the diffuse irradiance. The drop in the signal as the shadow passes over is equal to the direct irradiance times the cosine of the zenith angle. The zenith angle is derived from the instrument’s latitude, longitude and time. The total irradiance is measured when there is no shading.

As the photocell is sensitive to temperature variation, an ambient air temperature sensor is included, which permits to make temperature corrections to the photocell signal.

On-site radiation measuring is quite a long process if it is intended to achieve results that permit reliable interpolation to future radiation conditions. Even if exact results are achieved for a whole year,
they may permit only rough conclusions for other years, yet in many places solar radiation varies considerably over the years. For instance, the Deutsche Wetterdienst registered in the years 1937 to 1999 annual global irradiation values for Potsdam/Germany from 887 kWh/m²a to 1180 kWh/m²a. That means that the highest annual irradiation during that period was 33 percent higher than the lowest one. That’s why it is necessary to accomplish measurements during a longer time to get reliable results. Figure 52 shows the maximal variation of values determined during different periods (from 1 year to 20 years) in relation to the long term average from 1937 to 1999. In the case of Potsdam, the measurement period should comprise at least five years in order to limit the variation from the long term average to 7 per cent. In other words, average values for a five year period can vary up to 14 percent in relation to the average value for the following five years. Taking into consideration that an exact estimation of the solar resource is crucial for the economic calculation of solar energy projects, it is very important to count with long-term data.

Figure 52: Maximal variation of irradiation during periods from 1 to 20 years in relation to the long term average from 1937 to 1999, registered in Potsdam/Germany (source: Quaschning)

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48 Deutscher Wetterdienst is the German national meteorological service. It provides weather and climate information. See www.deutscher-wetterdienst.de.
5.2 Satellite measurement programmes

A sufficiently exact *in situ* measuring of solar irradiance is expensive and it takes a long time. It is expensive because of the required instruments and because of the necessary staff for the maintenance of the measurement station, and it takes a long time because, even if the measurement is realized over an entire year, the results may be little reliable. Additionally, ground measurements are local and do not provide information about larger regions.

A complementary possibility to acquire solar radiation data is the purchase of data from satellite measurement programmes. Generally, it is less expensive and covers larger time spans. Very important is that is also covers larger regions. It allows a quick comparison between different sites concerning their solar radiation potential. As the data of different sites are acquired by the same measurement instruments and procedures, a comparison should be very reliable. If an overview over a certain area is required, no interpolation processes are needed to determine local conditions at selected sites.

Several methods have been developed in the past two decades for estimating the solar irradiance on the basis of satellite data. One approved method is the so-called Heliosat method, which is used and permanently improved in the DLR project SOLEMII. This method distinguishes between two main factors of atmospheric influences on solar radiation: first, attenuation factors aside from cloudiness and, second, cloudiness. This method permits to account for global and direct radiation separately (and, consequently, also for diffuse radiation). Accordingly, it is accomplished in two major calculation steps. In a first step, the clear sky irradiance for a given location and time is calculated, i.e. a cloud free atmosphere is assumed and the influence of absorption and scattering at air molecules, ozone, water vapour and aerosols on the radiative transfer is determined. In a second step, a cloud index is derived from Meteosat images, which yields a cloud transmission and a clearness index. Cloud transmission and clearness index are used finally to determine global and direct irradiance on the basis of the clear sky global and direct irradiance. The calculation process is represented in the following schema (figure 53). The upper left field corresponds to the determination of the global and direct irradiance at clear sky conditions (step 1). The right field corresponds to the determination of the local cloudiness (step 2) and the lower left field to the final determination of average values of global and direct irradiance (and consequent irradiation values for given time periods).

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49 See Renne et al. 1999.
50 See DLR 2008, www.solemi.de
In the first step, the clear sky global irradiance on a horizontal surface $G_{\text{clear}}$ is determined as the sum of the clear sky direct irradiance $G_{\text{d,clear}}$ on the horizontal surface and the clear sky diffuse irradiance $G_{\text{b,clear}}$. $G_{\text{b,clear}}$, on its part, is clear sky direct normal irradiance $G_{\text{bn,clear}}$ multiplied by the cosine of the incident angle $\theta_z$:

$$G_{\text{clear}} = G_{\text{bn,clear}} \cos \theta_z + G_{\text{d,clear}} \quad (65)$$

In order to ascertain the clear sky global irradiance, thus, clear sky direct normal irradiance and clear sky diffuse irradiance have to be determined.

Clear sky direct normal irradiance $G_{\text{bn,clear}}$ is calculated by the application of different transmission factors to the irradiance on top of the atmosphere, i.e. on the solar constant $G_{sc}$. Each of these transmission factors corresponds to a certain atmospheric radiation attenuating process. The model distinguishes between $\tau_{rs}$ due to Rayleigh scattering, $\tau_o$ due to absorption by ozone, $\tau_g$ due to absorption by uniformly mixed gases (N₂, O₂, CO₂, CH₄ etc.), $\tau_w$ due to absorption by water vapour and $\tau_a$ due to absorption and scattering by aerosol particles. Taking into account these different transmission factors, $G_{\text{bn,clear}}$ is calculated as follows:

$$G_{\text{bn,clear}} = G_{sc} \tau_{rs} \tau_o \tau_g \tau_w \tau_a \quad (66)$$

$\tau_{rs}$ and $\tau_g$ are considered as depending exclusively on the air mass. That means, as the air mass depends just on the incident angle $\theta_z$, no further data are required besides the incident angle. $\theta_z$ is the only determinant of $\tau_{rs}$ and $\tau_g$.

In the case of $\tau_o$, $\tau_w$ and $\tau_a$, the situation is different. In order to determine these transmission factors, information about ozone distribution, water vapour and aerosol content in the atmosphere is needed:

- Ozone data are acquired from the NASA Ozone Monitoring Instrument (OMI), a satellite instrument for measuring ozone values. It provides global measurements of total column ozone on a daily basis. SOLEMI uses monthly data sets because the variability of the ozone
column depends, besides on geographical latitude, on the time of the year. The influence of ozone on the ground irradiance is relatively small (because of the small absorption range at wavelengths under 0.3 μm).

- Exact water vapour data are very important for the estimation of the ground irradiance, because water vapour has a larger influence on ground irradiance than ozone. Water vapour has important absorption ranges in the thermal spectrum. Data are acquired from the NCEP/NCAR-Reanalysis program (NCEP: National Center for Environmental Prediction (U.S.), NCAR: National Centre for Atmospheric Research (U.S.)). The objective of this program is to make available a continually updating data set that represents the state of the Earth’s atmosphere. It uses observation data and a global climate model that allows analyses back to 1948. The data were evaluated by a comparison with NVAP data (NVAP: NASA Water Vapor Project) and showed a correlation above 0.9 for most regions of the world.

- Aerosols have the strongest influence on clear sky irradiance, because of both absorption and scattering processes. SOLEMI compared several available aerosol data sets with data sets from different ground measurement networks (for instance Deutscher Wetterdienst (Germany), BSRN (Baseline Surface Radiation Network) by the World Climate Research Programme etc.). For direct normal irradiance as well as for global irradiance, the result of the comparison was that the NASA/GISS data sets (NASA Goddard Institute for Space Studies) coincide best with the ground measurements at present. Consequently, SOLEMI uses these data.

With these data sets, all transmission factors can be calculated according to given formulae (taking into consideration always the local and momentary air mass), and the clear sky direct irradiance can be determined.

Diffuse irradiance also depends on the different mentioned extinction processes. Anyway, the existence of diffuse radiation is just another result of these processes, besides the attenuation of direct radiation. However, there are two additional parameters that have to be taken into account for the determination of diffuse irradiance. As diffuse radiation is produced partially by reflection processes between the Earth’s surface and the atmosphere, diffuse radiation also depends on the ground albedo and the atmospheric albedo. The radiation reflected by the Earth’s surface may be backscattered or reflected back to the Earth and so contribute to the diffuse radiation. That’s why reflection processes have to be taken into consideration for the determination of ground irradiance. Altogether, the model distinguishes between three components of diffuse radiation: diffuse radiation due to Rayleigh scattering, diffuse radiation due to aerosol scattering and diffuse radiation due to multiple reflections between terrestrial surface and atmosphere. The sum of these three components yields the total diffuse irradiance. And the sum of total diffuse irradiance and horizontal direct irradiance is the horizontal clear sky global irradiance. The determination of this sum completes step 1 of the Heliosat method.

Step 2 in the procedure consists of the consideration of cloudiness. Clouds have the largest influence on atmospheric radiative transfer, so it is very important to have reliable data about cloudiness at a given place. These data are acquired by using Heliosat images. i.e. images acquired by meteorological geostationary satellites.\(^{51}\) By means of these images, different albedo values at a certain geographical point at different times can be registered. Differences in the observed albedos are correlated with different cloud covers, because the albedo seen from the satellite is higher for overcast conditions than for clear sky conditions. The registered albedo, which will be between the highest value for completely overcast sky and the lowest value for completely clear sky, is a measure, then, for the cloudiness at a given place and a given time.

\(^{51}\) The method was proposed originally by Cano (see Cano et al. 1986) and subsequently developed and improved by different authors (see especially Rigollier et al. 2004).
More precisely, the procedure is the following: Actual albedo values $\rho$ are derived from satellite images by evaluating brightness values. Data series about the measured albedo values for a given place are used to determine the local ground albedo $\rho_{\text{min}}$, which is identified as the minimal reflectance shown by the images. The ground albedo has to be identified for every site individually, because it varies in accordance with surface conditions. The ground albedo is the minimal possible reflectance. That means that the existence of clouds increases the reflectance in any case. The identification of reflectance values equal to the ground albedo is assumed, then, to correspond to clear sky conditions. The maximal albedo $\rho_{\text{max}}$, which corresponds to totally overcast sky, does not need to be identified individually at every site, because it does not depend on ground conditions. It depends exclusively on the incident angle. The actual reflectance $\rho$ at a given place and time must be between the local minimal reflectance (ground albedo) $\rho_{\text{min}}$ and the general maximal reflectance $\rho_{\text{max}}$. The cloud index $n$ indicates the value of $\rho$ in relation to $\rho_{\text{min}}$ and $\rho_{\text{max}}$. It is defined as follows:

$$n = \frac{\rho - \rho_{\text{min}}}{\rho_{\text{max}} - \rho_{\text{min}}}$$

(67)

$n$ is zero for clear sky ($\rho = \rho_{\text{min}}$) and 1 for overcast conditions ($\rho = \rho_{\text{max}}$).

On the basis of known clear sky direct and global irradiance, which are computed in step 1, the cloud index serves now for the determination of average values of direct and global irradiance. In order to calculate direct irradiance, the effective cloud transmission $\tau$ is introduced as the ratio of actual direct irradiance to clear sky direct irradiance. The direct irradiance $G_b$ is, then:

$$G_b = \tau \cdot G_{b,\text{clear}}$$

(68)

$\tau$ is related to $n$ according to $\tau = e^{-10n}$, so that all variables are known to determine $G_b$.

In order to calculate global irradiance, the clearness index $k_t$ is introduced as the ratio of actual global irradiance to clear sky global irradiance. The global irradiance $G$ is, then:

$$G = k_t \cdot G_{\text{clear}}$$

(69)

The empirically identified relation between $k_t$ and $n$ shows a simple linear characteristic for large ranges of $n$.

These irradiance data can be used subsequently to calculate temporal and spatial average values. Finally, global and direct irradiation data for given time periods can be made available. The values can be graphically represented as it is done in the following example, which shows the calculated irradiation for Saudi Arabia for July 2000:

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52 As the ground albedo of a given place may change, for instance due to changing vegetation conditions, ground albedo maps have to be computed repeatedly at regular intervals. SOLEMI uses monthly renewed values.

53 Taylor and Stowe (1984) indicate the following value: $\rho_{\text{max}} = 0.78 - 0.13(1 - e^{-4\cos(\theta_0)5})$.

54 For different ranges of $n$, $k_t$ is determined the following way: for $n \leq -0.2$, $k_t = 1.2$, for $-0.2 \leq n \leq 0.8$, $k_t = 1 - n$, for $0.8 \leq n \leq 1.1$ and for $1.1 \leq n$, $k_t = 0.05$. (Generally, $n$ varies between 0 (clear sky) and 1 (overcast conditions). However, values below 0 or above 1 may be registered. Values of $n$ below 0 occur typically at high incident angles on surfaces that don’t possess cloud cover but that are overshadowed from distant clouds. Values over 1 are registered occasionally when cloud reflectance exceeds the generally accepted maximum value.)
The satellites that are used for such measurements are geostationary satellites, i.e. satellites that have a fixed position in relation to the Earth. They are above the equator at a distance of about 36,000 km. Meteosat 7 and Meteosat 8 and 9 are the geostationary satellites that are relevant for Europe and MENA.

- fixed position in relation to Earth
- operating altitude: 36,000 km
- positioned above the equator
5.3 Available radiation data bases

There are several radiation data bases, which can be classified according to their data sources and the data processing. Some data bases are ground measurement data collections, others are based on satellite data, and, finally, there are data bases that process data (ground measurement and possibly also satellite data) in a spatial interpolation. Interpolation of ground measurement data between two or more ground measurement stations can be deployed when measurement stations are near to the location or site of interest. The interpolation may reach a higher accuracy than satellite data. In the following important radiation data bases are enumerated:

Ground measurement data sources:
- World Radiation Data Centre (WRDC)
- Global Energy Balance Archive (GEBA)
- Basline Surface Radiation Network (BSRN)
- International Daylight Measurement Programme (IDMP)

Data bases based on satellite information:
- NASA’s Surface Meteorology and Solar Energy (SSE)
- HelioClim
- Satel-Light
- SOLEMI

Data sources that combine ground measurement data with spatial interpolation (possibly including additional satellite data) are:
- Meteonorm
- European Solar Radiation Atlas (ESRA)
- Eosweb
- SoDa Service
- PVGIS
- Test-Reference-Year
- RETScreen

The following table shows the meteorological data provided by Meteonorm for Farafra/Egypt.
This chapter explained the energy source of CSP plants, solar radiation, and introduced some aspects of radiation concentration. The next chapters will explain in a more detailed way how the solar radiation is used in CSP plants in order to generate electricity.
Reference List


Questions and Exercises

Questions

1. Which two elements are the main components the Sun consists of? By which physical process the Sun generates the energy? What does the Sun loose through the generation of energy?

2. How does the radiant power of a thermal radiator vary if its temperature rises from 0°C to 273.15°C?

3. Rayleigh-Scattering is responsible for the blue colour of the clear sky. What is another colour effect of Rayleigh-Scattering at sunrise and at sunset and how can we explain this effect?

4. Why do we use Air Mass in order to indicate the optical way length through the atmosphere and not something like an “Air Distance”?

5. How does direct radiation vary (qualitatively) with altitude (all other things being equal) and how does global radiation vary with altitude?

6. Take a location at 30° N. On 21st of June at noon, is the irradiance due to direct radiation higher on a plane laying horizontally or on a plane facing to the south with a slope angle of 45°?

7. Which are the respective advantages of north-south and east-west alignment of parabolic troughs? Consider latitudes of about 30°.

8. What is the general physical reason why it is impossible for CSP systems to reach any concentration ratio?

9. What general kind of concentration geometry should we use if we want to maximize the collector temperature?

10. Why do we concentrate solar radiation? Why is it not always sufficient to harvest non-concentrated solar radiation?

11. a) What are the two main thermal loss mechanisms at the receivers of CSP systems? b) By means of which measures it is possible to reduce the these losses and rise, hence, the temperature of a given absorber in a concentrating system?

12. Why do we need satellite radiation measurement as well as ground measurement in order to realize site assessments? In which sense do they complement each other?

13. You want to make ground measurements of the solar radiation in a remote place where permanent surveillance is difficult and expensive. Which kind of measurement instruments do you chose? Which other instruments could you use if surveillance was easy and cheaper?
Answers

1. Hydrogen and Helium are the main components. Solar energy is generated by fusion of hydrogen into helium. Energy generation by nuclear fusion in the Sun is correlated to a permanent mass defect of the Sun.

2. As the temperature doubles (in the Kelvin scale), the radiant power octuplicates.

3. The orange or red colour of the Sun is another effect. As the Sun light has a longer way through the atmosphere, the short wave light is scattered much more and the red/orange part of the spectrum (with a longer wave length) is overrepresented in the arriving radiation.

4. The atmosphere has different densities in different altitudes and the extinction effect of the atmosphere the radiation travels through does not only depend on the distance but also on the density. It is the quantity of air the light has to pass, not the distance, that determines the grade of radiation attenuation.

5. Direct radiation is higher at higher altitudes (all other things being equal) because of reduced extinction processes. Global radiation is also higher at higher altitudes. Although diffuse radiation may increase at lower altitudes, the total amount of radiation diminishes because of increasing backscattering to space and increasing absorption.

6. Solar zenith angle: \(30^\circ - 23.45^\circ = 6.55^\circ\)
   \[\rightarrow \text{incidence angle on a horizontal plane: } 6.55^\circ\]
   \[\text{incidence angle on a plane facing to the south with a slope angle of } 45^\circ: 45^\circ - 6.55^\circ = 38.45^\circ\]
   \[\rightarrow \text{irradiance due to direct radiation is higher at the horizontal plane (at the given location and at the indicated time)}\]

7. north-south:
   - quite equilibrate daily power curve
   - very low cosine losses in the morning and in the evening
   - total annual energy yield slightly higher

   east-west:
   - small collector adjustments during the day
   - no cosine losses at noon
   - smaller energy yield differences between summer and winter

8. The spread angle of the beam radiation limits the concentration ratio. Only perfect parallel radiation would permit to rise the concentration ratio at will. But, as any real radiation source has an extension, there is no perfect parallel radiation.

9. We should use point-concentrating systems.
10. High temperatures can be reached only with a sufficiently high radiation concentration. High temperatures are necessary for an efficient conversion of heat into mechanical energy. Non-concentrated radiation can be used for low-temperature applications.

11. a) convective losses and radiative losses
   b) transparent claddings reduce convective losses, selective radiation transmission can reduce additionally the radiative heat loss (for instance glass with high transmission coefficient for short wave radiation and low transmission coefficient for long wave radiation)
   c) selective coatings reduce radiative losses (high absorption coefficient for short wave radiation, low emission coefficient for long wave radiation)

12. Satellite measurement and ground measurement are complementary in the sense that just one of them (satellite measurement) comprises large areas and permits the comparison between many possible sites and just one of them (ground measurement) achieves a sufficiently high exactness for a sufficiently good CSP performance assessment. Both are important: Satellite measurement permits the solar resource assessment for large areas and ground measurement is apt to determine very exactly the solar radiation situation at the local level. So, one possible strategy is to apply satellite measurement to preselect appropriate sites and to apply ground measurement to study more exactly some of these preselected sites.

13. In this case it is better to use a rotating shadow band pyranometer, which does not need daily cleaning. If surveillance was easy and cheaper, then pyrheliometer or pyranometer in combination with pyranometers with shadow elements (shadow ring, shadow ball) can be used.
Exercises

Exercise 1

At a location near Djelfa/Algeria with the coordinates 34°33’ N, 3° 15’ E a CSP plant is planned. You have to program the Sun tracking unit. Therefore, you need some exact Sun position indications. Algeria is within the Central European Time zone with the Prime Meridian (longitude 0°) as the reference longitude.

a) What is the position of the Sun in the horizontal coordinate system on 10th of February at 16:30 Local Standard Time?

b) At what time (Local Standard Time) the Sun crosses the local meridian on 1st of October?

Exercise 2

A businessman offers his old stone quarry near Ouarzazate/Morocco (30°56’ N, 6°55’W) as a location for the construction of a CSP plant. The stone quarry has approximately a circular form, as to be seen in the following figure.

The location would have certain advantages: Land costs are low, underground water for cooling is easily accessible (because of the near Atlas mountains) and the quarry has an ideal plane horizontal surface, which would reduce construction costs compared to many alternative sites.

Unfortunately, the quarry has the disadvantage that the planned solar field would be shaded in the early morning and in the late evening. In a first evaluation of the situation, you want to calculate the duration of shading in the morning (after sunrise) and in the evening (until sunset). For simplicity, you decide to consider a circular solar field as indicated in the figure. You decide to calculate the shading on 21st of December and on 21st of June. Indicate the duration (in solar time) of complete shading as well as of partial shading.
Exercise 3

A project developer plans a CSP plant near Zarzis/Tunisia. The coordinates are $33^\circ30'\ N$ and $11^\circ7'\ E$. He wants to get an approximate calculation of the irradiation at this location.

a) You decide to consider irradiation on summer solstice. At what time are sunrise and sunset? Indicate the day length. (All in solar time)

b) What is the solar zenith angle at 12:00 solar time? Calculate the solar zenith angle additionally for 9:00 and 15:00 solar time and plot the approximate solar zenith angle run from sunrise to sunset.

c) Calculate approximately the irradiation on the horizontal surface during one day at summer solstice. Measurements have shown that the turbidity factor in summer at the indicated location is around 4.5.

i) Indicate the general mathematical expression for the irradiation, taking into consideration that it is the integral of irradiance over time. Take into consideration that only direct radiation is useful for CSP. Express the incidence angle on the horizontal plane as a function of latitude, declination and hour angle.

ii) Approximate irradiation performing a piecewise integration of the integral from (i) taking average values of $\theta_z$ for the time lapses between sunrise and 9:00, 9:00 and 12:00, 12:00 and 15:00 and between 15:00 and sunset. (Remember that $d\omega = \frac{2\pi}{24} dt$, expressing angles in [rad] and time in [h]).

Exercise 4

A solar trough power plant will be built near Cairo ($30^\circ\ N$, $31^\circ50'\ E$). You have to design the Sun tracking system for the solar troughs, which have a north-south alignment. You want to approximate the angular velocity of the troughs in order to choose motor and gear. Therefore you want to estimate the angular velocity of the troughs in the early morning (respectively late evening) and at noon and you want to distinguish between winter and summer.

a) Approximate the angular velocity between 5:30 and 6:00 and 11:30 and 12:00 (solar time) on 21st of June and between 7:30 and 8:00 and 11:30 and 12:00 an 21st of December.

b) Is it possible to derive from the result of (a) any general qualitative statement about differences of the angular velocity between summer and winter and between noon and early morning (respectively late evening)? If there are such generalizations, for which part of the world are they valid?
Solutions

Exercise 1

a)

i) Determination of solar coordinates in the equatorial system:

a) declination:

\[ \delta = 23.45^\circ \sin \left(360^\circ \frac{284 + DoY}{365}\right) \]

\[ DoY = 41 \]

\[ \delta = -14.90^\circ \]

b) hour angle:

Conversion of Local Standard Time into Solar Time:

\[ t_{so} = (L_r - L_{loc}) \left(4 \frac{\text{min}}{1^\circ}\right) + E + t_{std} \]

\[ L_r = 0^\circ, \ L_{loc} = -3.25^\circ \]

\[ E = 229.2 \left(0.000075 + 0.001868 \cos d - 0.032077 \sin d \right) \min 
- 0.014615 \cos 2d - 0.0409 \sin 2d \]

\[ d = 360^\circ \cdot (DoY - 1)/365 = 39.45^\circ \]

\[ E = -14.16 \text{min} \]

\[ t_{so} = (0^\circ + 3.25^\circ) \left(4 \frac{\text{min}}{1^\circ}\right) + (-14.16 \text{min}) + 16:30h = 16:28.84h \]

\[ \omega = (16:28.84h - 12:00h) \frac{1^\circ}{4 \text{min}} = 268.84 \text{min} \cdot \frac{1^\circ}{4 \text{min}} = 67.21^\circ \]

ii) Determination of solar coordinates in the horizontal system:

a) solar altitude angle:

\[ \alpha_s = \arcsin (\cos \Phi \cos \delta \cos \omega + \sin \Phi \sin \delta) \]

\[ = \arcsin \left(\cos 34.55^\circ \cos (-14.9^\circ) \cos 67.21^\circ + \sin 34.55^\circ \sin (-14.9^\circ)\right) \]

\[ \alpha_s = 9.35^\circ \]

b) solar azimuth angle:
\[ y_s = \text{sign}(\omega) \left| \cos^{-1} \left( \frac{\sin(\delta) - \sin(\delta)}{\cos(\phi)} \right) \right| \]
\[ y_s = \arccos \left( \frac{\sin(9.35^\circ) \sin(34.55^\circ) - \sin(-14.9^\circ)}{\cos(9.35^\circ) \cos(34.55^\circ)} \right) \]
\[ y_s = 64.55^\circ \]

b)

\[ t_{std} = -(L_r - L_{loc}) \left( 4 \frac{\text{min}}{1^\circ} \right) - E + t_{so} \]
\[ t_{so} = 12:00\,\text{h} \]

DoY = 274

d = 360^\circ \cdot (274 - 1)/365 = 269.26^\circ

\[ E = 229.2 \left( 0.000075 + 0.001868 \cos d - 0.032077 \sin d \right) \min = 10.47\min \]
\[ L_r = 0^\circ, L_{loc} = -3.25^\circ \]
\[ t_{std} = -(0^\circ - (-3.25^\circ)) \left( 4 \frac{\text{min}}{1^\circ} \right) - 10.47\min + 12:00\,\text{h} = 11:48.72\,\text{h} \]
\[ t_{std} = 11:48h\,43s \]

Exercise 2

Because of symmetry the shading duration in the morning and in the evening is the same.

21\textsuperscript{st} of June:

sunrise:
\[ \cos \omega_{ks} = -\tan \delta \tan \Phi \]
\[ \delta = 23.45^\circ \]
\[ \Phi = 30.93^\circ \]
\[ \omega_{ks,1} = -105.07^\circ \]

first direct radiation:
\[ \sin \alpha_s = \sin \Phi \sin \delta + \cos \Phi \cos \delta \cos \omega \]
\[ \sin \alpha_s = \frac{50}{\sqrt{1250^2 + 50^2}} = 0.03997 \]
\[ \cos \omega = \frac{\cos \Phi \cos \delta}{\sin \alpha_s - \sin \Phi \sin \delta} \]
\[ \omega_1 = -102.07^\circ \]

end of shading:
\[ \sin \alpha_s = \sin \Phi \sin \delta + \cos \Phi \cos \delta \cos \omega \]
\[
\sin \alpha_s = \frac{50}{\sqrt{250^2 + 50^2}} = 0.19612
\]

\[
\omega_2 = -90.61^\circ
\]

duration total shading: \( t_{st} = \frac{4\text{min}}{1^\circ} \left( -102.07^\circ - (-105.07^\circ) \right) = 12\text{min} \)

duration partial shading: \( t_{sp} = \frac{4\text{min}}{1^\circ} \left( -90.61^\circ - (-102.07^\circ) \right) = 45.84\text{min} = 45\text{min}50\text{s} \)

**Total shading in the morning and in the evening of the 21st of June:** 12min each

**Partial shading in the morning and in the evening of the 21st of June:** 45min50s each

21st of December:

sunrise:

\[
\cos \omega_{ss} = -\tan \delta \tan \Phi
\]

\( \delta = -23.45^\circ \)

\( \Phi = 30.93^\circ \)

\( \omega_{ss,1} = -74.93^\circ \)

first direct radiation:

\[
\sin \alpha_s = \frac{50}{\sqrt{1250^2 + 50^2}} = 0.03997
\]

\[
\cos \omega = \frac{\sin \alpha_s - \sin \Phi \sin \delta}{\cos \Phi \cos \delta}
\]

\( \omega_1 = -71.90^\circ \)

end of shading:

\[
\sin \alpha_s = \sin \Phi \sin \delta + \cos \Phi \cos \delta \cos \omega
\]

\[
\sin \alpha_s = \frac{50}{\sqrt{250^2 + 50^2}} = 0.19612
\]

\( \omega_2 = -59.39^\circ \)

duration total shading: \( t_{st} = \frac{4\text{min}}{1^\circ} \left( -71.90^\circ - (-74.93^\circ) \right) = 12.12\text{min} = 12\text{min}7\text{s} \)

duration partial shading: \( t_{sp} = \frac{4\text{min}}{1^\circ} \left( -59.39^\circ - (-71.90^\circ) \right) = 50.04\text{min} = 50\text{min}2\text{s} \)

**Total shading in the morning and in the evening of the 21st of December:** 12min 7s each

**Partial shading in the morning and in the evening of the 21st of December:** 50min 2s each
Exercise 3

a) \[ \cos \omega_{ss} = -\tan \delta \tan \Phi \]
\[ \delta = 23.45^\circ \]
\[ \Phi = 33.5^\circ \]
\[ \omega_{ss} = 106.7^\circ \]
\[ \omega_{ss,1} = -106.7^\circ \]
\[ t_{ss,1} = 12:00 - \frac{106.7^\circ}{15^\circ} \text{[h]} = 4.89 \text{h} = 4:53 \text{h} \]
\[ t_{ss,2} = 12:00 + \frac{106.7^\circ}{15^\circ} \text{[h]} = 19.11 \text{h} = 19:07 \text{h} \]
\[ \text{day length} = 19.11 \text{h} - 4.89 \text{h} = 14.22 \text{h} = 14 \text{h} 13 \text{min} \]

b) at 12:00:
\[ \theta_z = \Phi - \delta = 10.05^\circ \]

at 9:00 and at 15:00:
\[ \cos \theta_z = \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega \]
\[ \omega = \pm 3 \text{h} \cdot \frac{15^\circ}{1 \text{h}} = \pm 45^\circ \]
\[ \cos \theta_z = 0.7606 \]
\[ \theta_z = 40.5^\circ \]
i) \[
\frac{dH}{dt} = G_b = G_{sc} e^{-\frac{T}{0.9+9.4 \cos \theta}} \cos \theta
\]
\[= G_{sc} e^{-\frac{T}{0.9+9.4 \cos \theta}} \left( \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega \right) \]
\[dH = G_{sc} e^{-\frac{T}{0.9+9.4 \cos \theta}} \left( \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega \right) dt \]
\[H = \int_t G_{sc} e^{-\frac{T}{0.9+9.4 \cos \theta(t)}} \left( \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega(t) \right) dt \]

ii) \[
H = \int_t G_{sc} e^{-\frac{T}{0.9+9.4 \cos \theta}} \left( \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega \right) dt
\]
\[= \int_0^{\frac{12}{\pi}} G_{sc} e^{-\frac{T}{0.9+9.4 \cos \theta}} \left( \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega \right) d\omega \]

piecewise integration with average values of \(\theta_z\):

from sunrise (\(\omega = -106.7^\circ = -1.8623\)) to 9:00 (\(\omega = -\pi/4\)): \(\overline{\theta_z} = 64.75^\circ\)

\[H_1 = \frac{12}{\pi} 1367 \text{ Wh/m}^2 e^{-\frac{4.5}{0.9+9.4 \cos 64.75}} \int_{-\pi/4}^{-1.8623} \left( \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega \right) d\omega \]
\[H_1 = 2312 \text{ Wh/m}^2 \left[ -\frac{\pi}{4} \sin \delta \sin \Phi + \cos \delta \cos \Phi \sin \left( -\frac{\pi}{4} \right) \right] + 1.8623 \cdot \sin \delta \sin \Phi \sin (1.8623) \]
\[H_1 = 894 \text{ Wh/m}^2 \]

from 9:00 (\(\omega = -\pi/4\)) to 12:00 (\(\omega = 0\)): \(\overline{\theta_z} = 25.275^\circ\)

\[H_2 = \frac{12}{\pi} 1367 \text{ Wh/m}^2 e^{-\frac{4.5}{0.9+9.4 \cos 25.275}} \int_{-\pi/4}^{0} \left( \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \omega \right) d\omega \]
\[H_2 = 3235 \text{ Wh/m}^2 \left[ \frac{\pi}{4} \sin \delta \sin \Phi - \cos \delta \cos \Phi \sin \left( \frac{\pi}{4} \right) \right] = 2308 \text{ Wh/m}^2 \]

from 12:00 to 15:00: \(H_3 = H_2 = 2308 \text{ Wh/m}^2 \)

from 15:00 to sunset: \(H_4 = H_1 = 894 \text{ Wh/m}^2 \)

total: \(H = H_1 + H_2 + H_3 + H_4 = 6404 \text{ Wh/m}^2 \)
Exercise 4

a) Linear approximation: $\dot{s} \approx \frac{\Delta s}{\Delta t}$

\[
\tan s = \tan \theta_2 \cos (\gamma - \gamma_2)
\]

\[
\Delta s = \arctan \left( \frac{\tan \theta_2 \cos (\gamma_2 - \gamma_{2,2})}{\tan \theta_1 \cos (\gamma_1 - \gamma_{1,1})} \right)
\]

$\phi = 30^\circ$

21st of June:

$\delta = 23.45^\circ$

5:30-6:00:

$\gamma = -90^\circ$

$\theta_{x,5:30} = \arccos \left( \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega \right)$

$\theta_{x,6:00} = \arccos \left( \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega \right)$

$\gamma_{5,30} = -\left| \arccos \left( \frac{\cos \theta_{x,5:30} \sin \phi - \sin \delta}{\sin \theta_{x,5:30} \cos \phi} \right) \right| = -113.97^\circ$

$\gamma_{5,6:00} = -\left| \arccos \left( \frac{\cos \theta_{x,6:00} \sin \phi - \sin \delta}{\sin \theta_{x,6:00} \cos \phi} \right) \right| = -110.59^\circ$

\[
\frac{\Delta s}{\Delta t} = \frac{\arctan \left( \frac{\tan 23.45^\circ \sin 30^\circ + \cos 23.45^\circ \cos 30^\circ \cos \left(-90^\circ\right)}{0.5h} \right) - \arctan \left( \frac{\tan 23.45^\circ \sin 30^\circ + \cos 23.45^\circ \cos 30^\circ \cos \left(-113.97^\circ\right)}{0.5h} \right)}{0.5h} = 12.51^\circ / h = 12.51'' / s
\]

11:30-12:00

$\gamma = -90^\circ$

$\theta_{x,11:30} = \arccos \left( \sin 23.45^\circ \sin 30^\circ + \cos 23.45^\circ \cos 30^\circ \cos \left(-7.5^\circ\right) \right) = 9.36^\circ$

$\theta_{x,12:00} = \arccos \left( \sin 23.45^\circ \sin 30^\circ + \cos 23.45^\circ \cos 30^\circ \cos 0^\circ \right) = 6.55^\circ$

$\gamma_{5,11:30} = -\left| \arccos \left( \frac{\cos 9.36^\circ \sin 30^\circ + \sin 23.45^\circ}{\sin 9.36^\circ \cos 30^\circ} \right) \right| = -47.37^\circ$

$\gamma_{5,12:00} = 0^\circ$

\[
\frac{\Delta s}{\Delta t} = \frac{\arctan \left( \frac{\tan 6.55^\circ \cos \left(-90^\circ\right)}{0.5h} \right) - \arctan \left( \frac{\tan 9.36^\circ \cos \left(-47.37^\circ\right)}{0.5h} \right)}{0.5h} = 13.83^\circ / h = 13.83'' / s
\]

21st of December:

$\delta = -23.45^\circ$

7:30-8:00:
\[ \gamma = -90^\circ \]
\[ \theta_{z, 7.30} = \arccos(\sin(-23.45^\circ)\sin30^\circ + \cos(-23.45^\circ)\cos30^\circ\cos(-67.5^\circ)) = 83.97^\circ \]
\[ \theta_{z, 8.00} = \arccos(\sin(-23.45^\circ)\sin30^\circ + \cos(-23.45^\circ)\cos30^\circ\cos(-60^\circ)) = 78.56^\circ \]
\[ \gamma_{s, 7.30} = -\left| \arccos \left( \frac{\cos83.97^\circ\sin30^\circ - \sin(-23.45^\circ)}{\sin83.97^\circ\cos30^\circ} \right) \right| = 58.46^\circ \]
\[ \gamma_{s, 8.00} = -\left| \arccos \left( \frac{\cos78.56^\circ\sin30^\circ - \sin(-23.45^\circ)}{\sin78.56^\circ\cos30^\circ} \right) \right| = 54.15^\circ \]

\[ \frac{\Delta \gamma}{\Delta t} = \frac{\arctan[\tan78.56^\circ\cos(-90^\circ-(54.15^\circ))] - \arctan[\tan83.97^\circ\cos(-90^\circ-(58.46^\circ))]}{0.5h} \]
\[ = 13.90^\circ/h = 13.90''/s \]

11:30-12:00
\[ \gamma = -90^\circ \]
\[ \theta_{z, 11.30} = \arccos(\sin(-23.45^\circ)\sin30^\circ + \cos(-23.45^\circ)\cos30^\circ\cos(-7.5^\circ)) = 53.93^\circ \]
\[ \theta_{z, 12.00} = \arccos(\sin(-23.45^\circ)\sin30^\circ + \cos(-23.45^\circ)\cos30^\circ\cos0^\circ) = 53.45^\circ \]
\[ \gamma_{s, 11.30} = -\left| \arccos \left( \frac{\cos53.93^\circ\sin0^\circ - \sin(-23.45^\circ)}{\sin53.93^\circ\cos30^\circ} \right) \right| = 8.49^\circ \]
\[ \gamma_{s, 12.00} = 0^\circ \]
\[ \frac{\Delta \gamma}{\Delta t} = \frac{\arctan[\tan53.45^\circ\cos(-90^\circ-(0^\circ))] - \arctan[\tan53.93^\circ\cos(-90^\circ-(8.49^\circ))]}{0.5h} \]
\[ = 22.92^\circ/h = 22.92''/s \]

b) The angular velocity is higher at noon than in the morning and in the evening, and the angular velocity at noon is higher in winter than in summer.
These statements are generally true for locations outside the tropics.